Consultant Designer
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كيف يتم أختيار نوع البلاطة في التصميم

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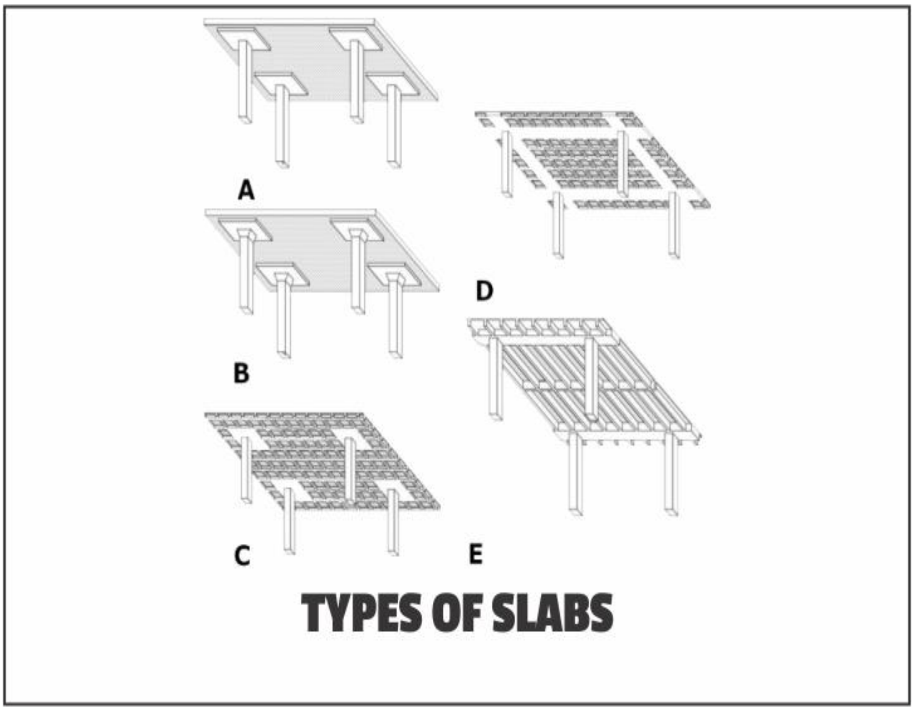
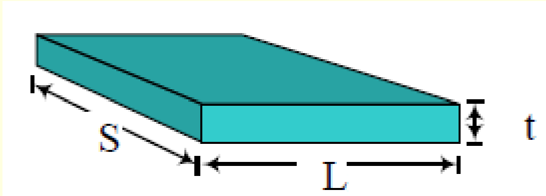


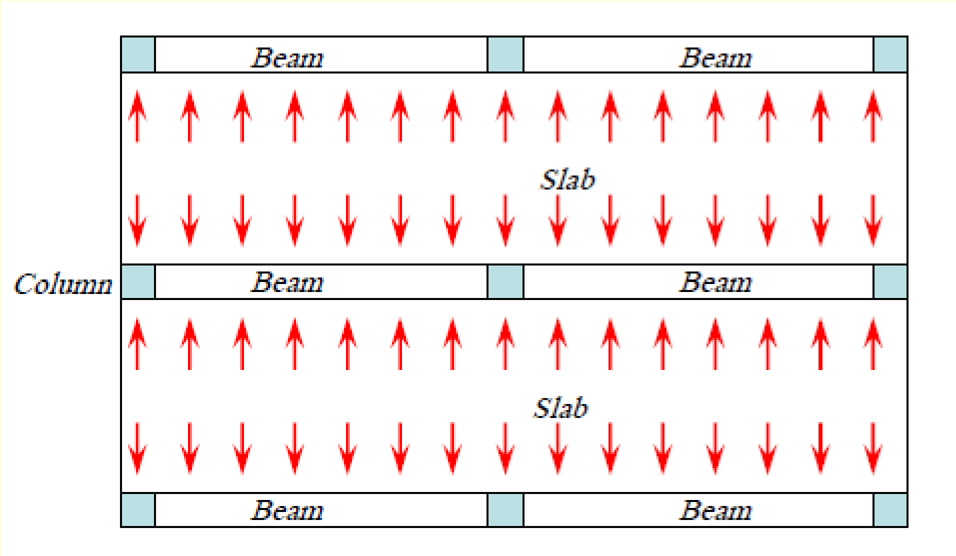
Introduction1

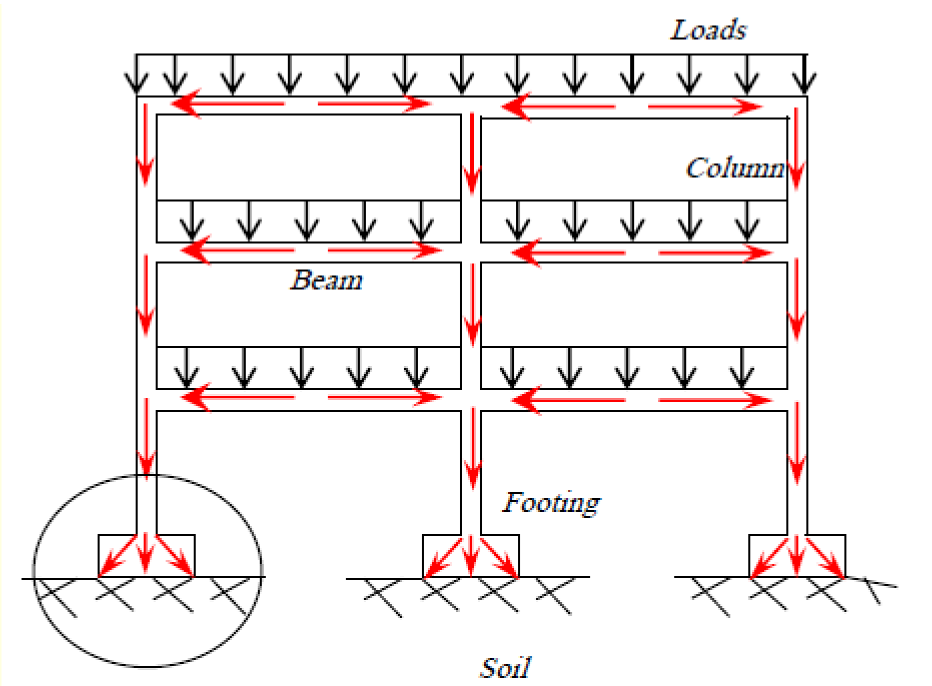
A slab is a structural element whose thickness is small compared to its own length and width.

$$t << L\ ,\ S$$



Slabs in buildings are usually used to transmit the loads on floors and roofs to the supporting beams.





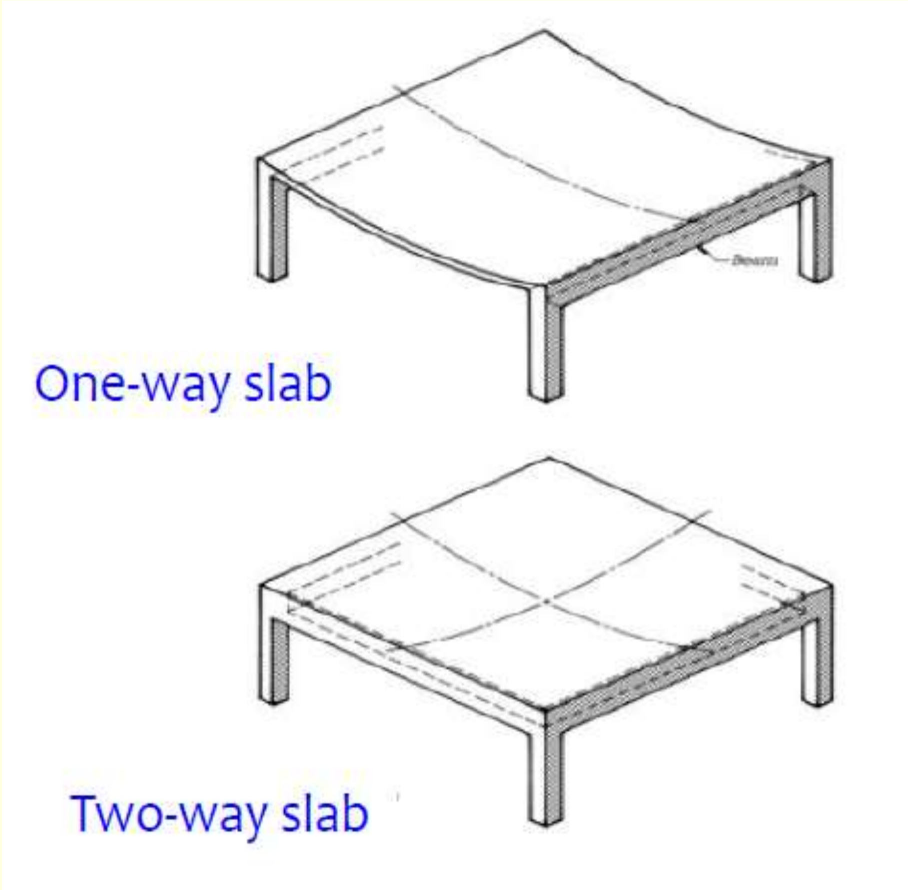
Slabs are flexural members. Their flexure strength requirement maybe expressed by:

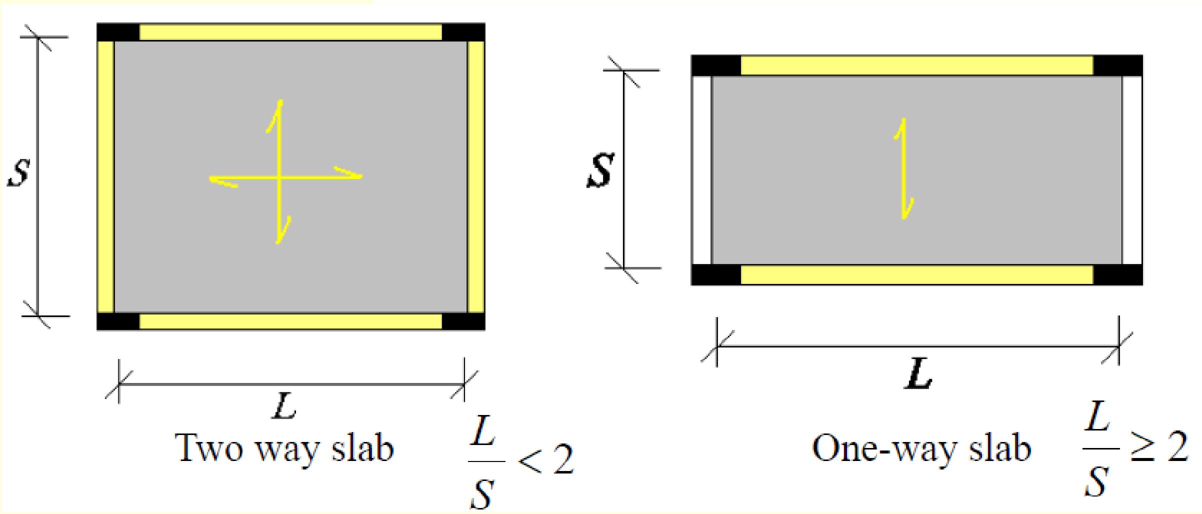
Mu ≤ΦMn **7.5.1.1ACI code318M-14**

Types of Slabs:

1- Solid slabs :- which are divided into

- One way solid slabs
- Two way solid slabs



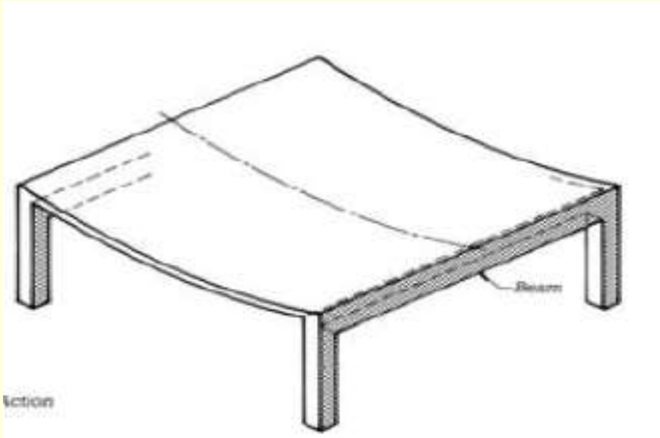


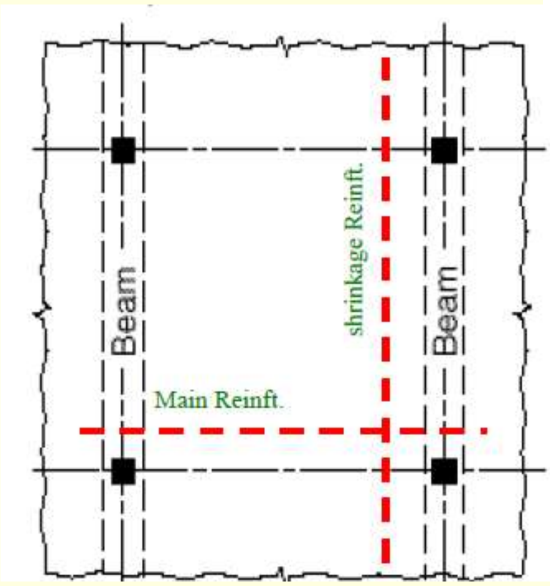
2- Ribbed slabs :- which are divided into

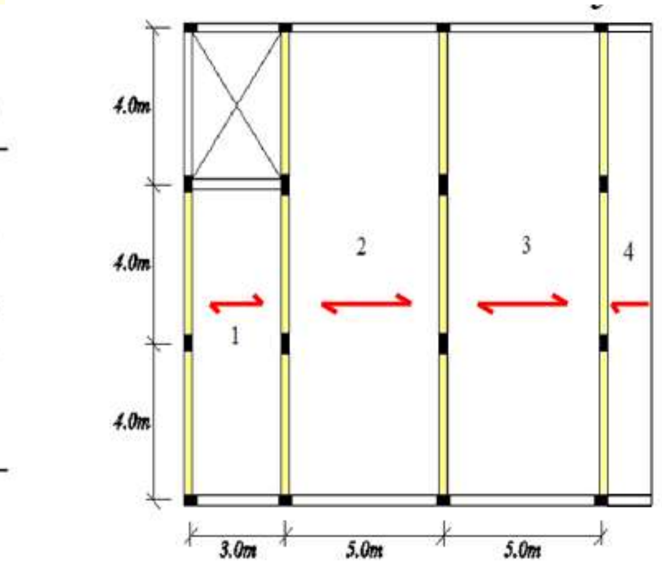
- One way ribbed slabs
- Two way ribbed slabs

One-way solid slabs

A one-way solid slab curves under loads in one direction only. Accordingly, slabs supported on two opposite sides only and slabs supported on all four sides, but L/S ≥ 2 are classified as one-way slabs. Main reinforcement is placed in the shorter direction, while the longer direction is provided with shrinkage reinforcement to limit cracking.

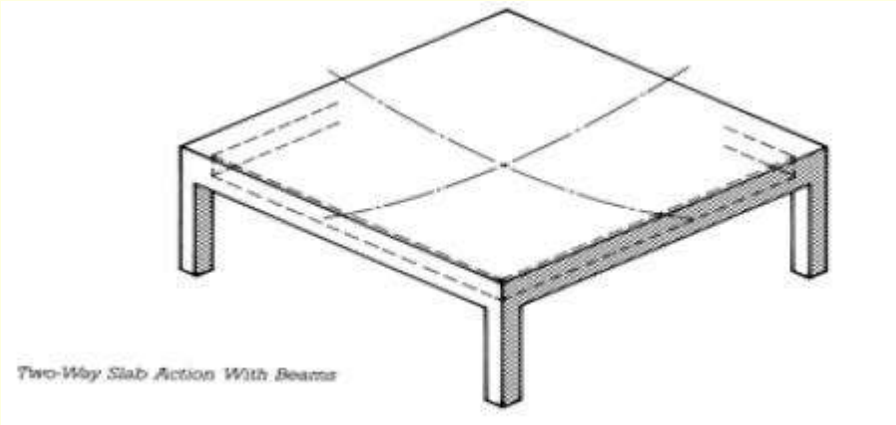


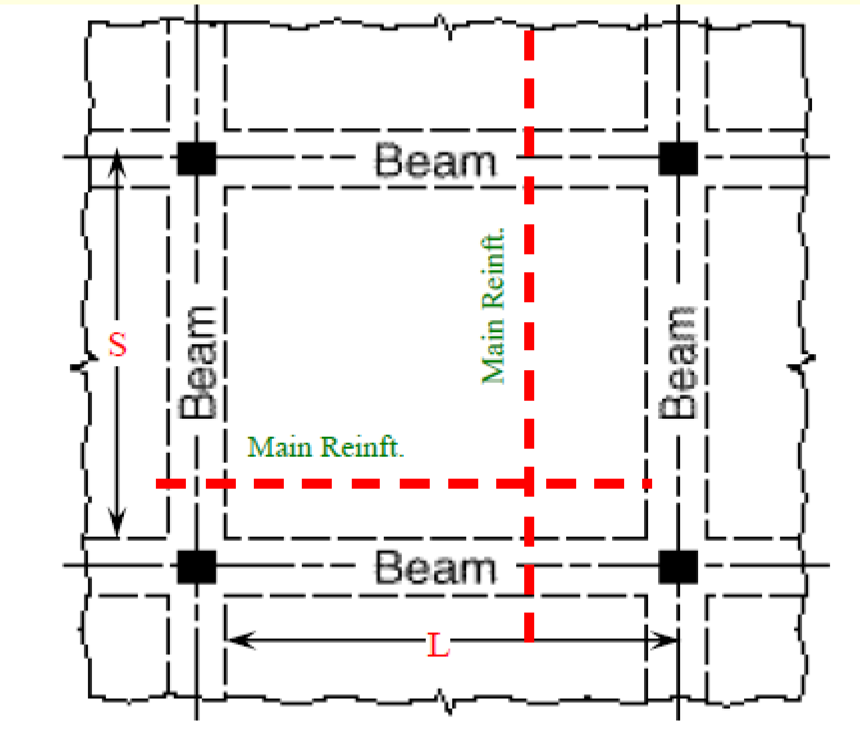




Two-way solid slabs

A two-way solid slab curves under loads in two directions. Accordingly slabs supported on all four sides, and L/S < 2 are classified as two-way slabs. Bending will take place in the two directions in a dish-like form. Accordingly, main reinforcement is required in the two directions.





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Minimum thickness of one way slabs

7.3.1 *Minimum slab thickness ACI 318M-14*

Table 7.3.1.1—Minimum thickness of solid nonprestressed one-way slabs

Support condition	Minimum h[1]
Simply supported	$\ell/20$
One end continuous	$\ell/24$
Both ends continuous	$\ell/28$
Cantilever	$\ell/10$

[1]Expression applicable for normalweight concrete and f_y = 420 MPa. For other cases, minimum h shall be modified in accordance with 7.3.1.1.1 through 7.3.1.1.3, as appropriate.

7.3.1.1.1 For ***fy*** other than 420 MPa, the expressions in Table 7.3.1.1 shall be multiplied by **(0.4 + *fy*/700)**.

Minimum Cover

20.6.1.3 *Specified concrete cover requirements ACI 318M-14*

Table 20.6.1.3.1—Specified concrete cover for cast-in-place nonprestressed concrete members

Concrete exposure	Member	Reinforcement	Specified cover, mm
Cast against and permanently in contact with ground	All	All	75
Exposed to weather or in contact with ground	All	No. 19 through No. 57 bars	50
		No. 16 bar, MW200 or MD200 wire, and smaller	40
Not exposed to weather or in contact with ground	Slabs, joists, and walls	No. 43 and No. 57 bars	40
		No. 36 bar and smaller	20
	Beams, columns, pedestals, and tension ties	Primary reinforcement, stirrups, ties, spirals, and hoops	40

U.S. rebar size chart

Imperial bar size	Metric size	Linear Mass Density		Nominal diameter		Nominal area	
		lb/ft	(kg/m)	(in)	(mm)	(in²)	(mm²)
#2	#6	0.167	0.249	0.250 = 2/8	6.35	0.05	32
#3	#10	0.376	0.561	0.375 = 3/8	9.525	0.11	71
#4	#13	0.668	0.996	0.500 = 4/8	12.7	0.20	129
#5	#16	1.043	1.556	0.625 = 5/8	15.875	0.31	200
#6	#19	1.502	2.24	0.750 = 6/8	19.05	0.44	284
#7	#22	2.044	3.049	0.875 = 7/8	22.225	0.60	387
#8	#25	2.670	3.982	1.000	25.4	0.79	509
#9	#29	3.400	5.071	1.128	28.65	1.00	645
#10	#32	4.303	6.418	1.270	32.26	1.27	819
#11	#36	5.313	7.924	1.410	35.81	1.56	1006
#14	#43	7.650	11.41	1.693	43	2.25	1452
#18	#57	13.60	20.284	2.257	57.3	4.00	2581
#18J		14.60	21.775	2.337	59.4	4.29	2678

Spacing of Reinforcement Bars

a- Flexural Reinforcement Bars 8.7.2.2 ACI 318-14

Flexural reinforcement is to be spaced not farther than three times the slab thickness (hs), nor farther apart than 45 cm, center-to-center.

$$S_{max} \leq the\ smaller\ of\ \frac{2h_s}{450mm}$$ for critical section

$$S_{max} \leq the\ smaller\ of\ \frac{3h_s}{450mm}$$ for other sections

b- Shrinkage Reinforcement Bars 24.4.3.3 ACI 318-14

Shrinkage reinforcement is to be spaced not farther than five times the slab thickness, nor farther apart than 45 cm, center-to-center.

$$S_{max} \leq the\ smaller\ of\ \frac{5h_s}{450mm}$$

Loads Assigned to Slabs

$$W_u = 1.2\ \text{D.L} + 1.6\ \text{L.L}$$

a- Dead Load (D.L):

1-Weight of slab covering materials
2-Equivalent partition weight
3- Own weight of slab

b- Live Load (L.L):

a- Dead Load (D.L)

1-Weight of slab covering materials =2.315 kN/m^2

Tiles (2.5cm thick) =0.025×23 kN/m^2
Cement mortar (2.5cm thick) =0.025×21 kN/m^2
Sand (5.0cm thick) =0.05×18 kN/m^2
Plaster (1.5cm thick) =0.015×21 kN/m^2

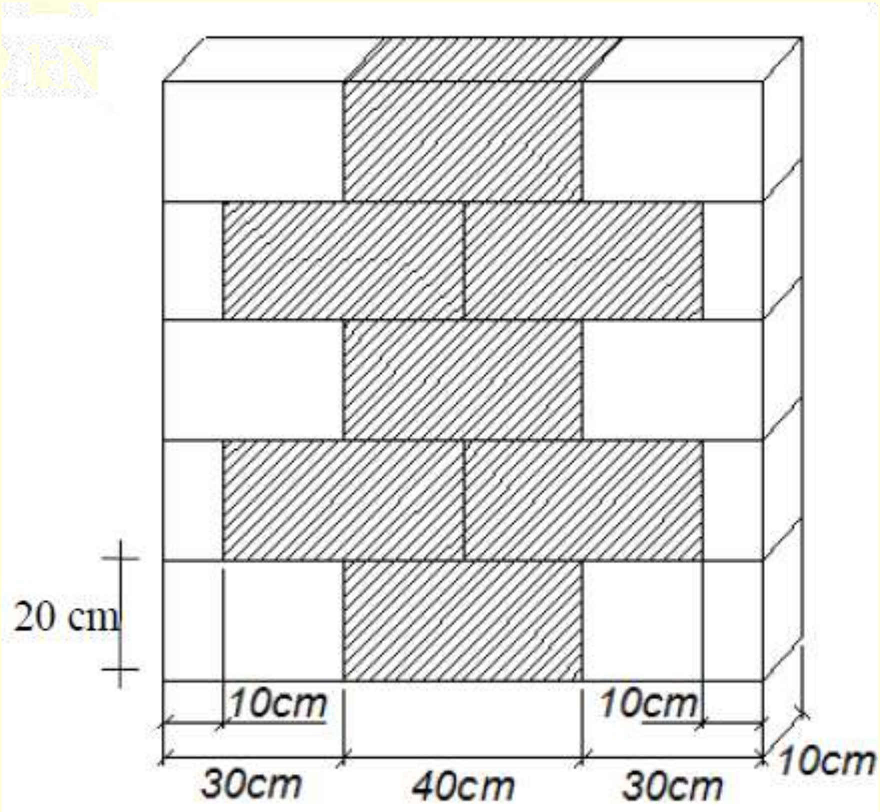
2-Equivalent partition weight

This load is usually taken as the weight of all walls (weight of 1m span of wall × total spans of all walls) carried by the slab divided by the floor area and treated as a dead load rather than a live load.

To calculate the weight of 1m span of wall:

Each $1m^2$ surface of wall contains 12.5 blocks.
A block with thickness 10cm weighs 10 kg = 0.1 kN.
A block with thickness 20cm weighs 20 kg = 0.2 kN.
Each face of 1m2 surface has 30kg plaster.
Load / $1m^2$ surface for 10 cm block $=12.5 \times 10 + 2\times30 = 185$ $Kg/m^2 = 1.85 kN/m^2$
Load / 1m2 surface for 20 cm block $=12.5 \times 20 + 2\times30 = 310$ $kg/m^2 = 3.1$ kN/m^2

Weight of 1 m span of wall with height 3 m:
For 10 cm block wt. = 1.85 $KN/m^2 \times 3 = 5.6 kN/m$.
For 20 cm block wt. = 3.1 $KN/m^2 \times 3 = 9.3$ kN/m.



3- Own weight of slab

Own weight of solid slab (per unit $1m^2$) = $\gamma * h = 25h$ kN/m^2

b- Live Load (L.L):

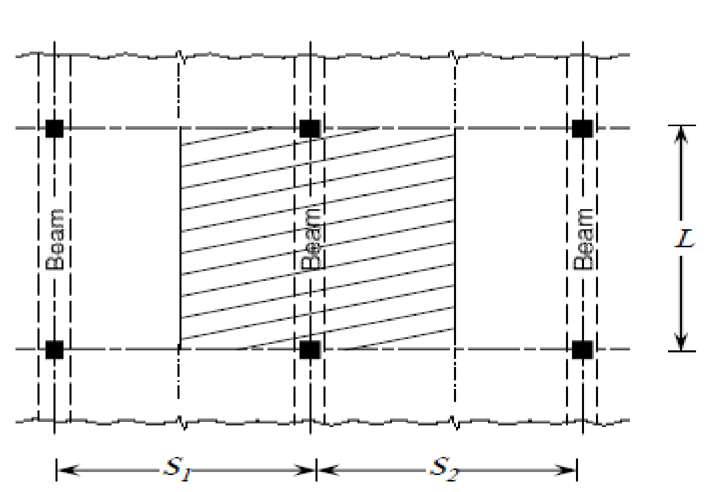
It depends on the function for which the floor is constructed.

Type of Use	Uniform Live Load kN/m^2
Residential	2
Residential balconies	3
Computer use	5
Offices	2
Warehouses	
▪ Light storage	6
▪ Heavy Storage	12
Schools	
▪ Classrooms	2
Libraries	
▪ rooms	3
▪ Stack rooms	6
Hospitals	2
Assembly Halls	
▪ Fixed seating	2.5
▪ Movable seating	5
Garages (cars)	2.5
Stores	
▪ Retail	4
▪ wholesale	5
Exit facilities	5
Manufacturing	
▪ Light	4
▪ Heavy	6

Loads Assigned to Beams

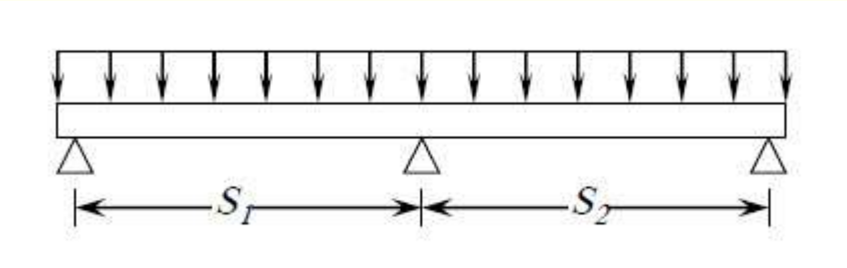
Beams are usually designed to carry the following loads:

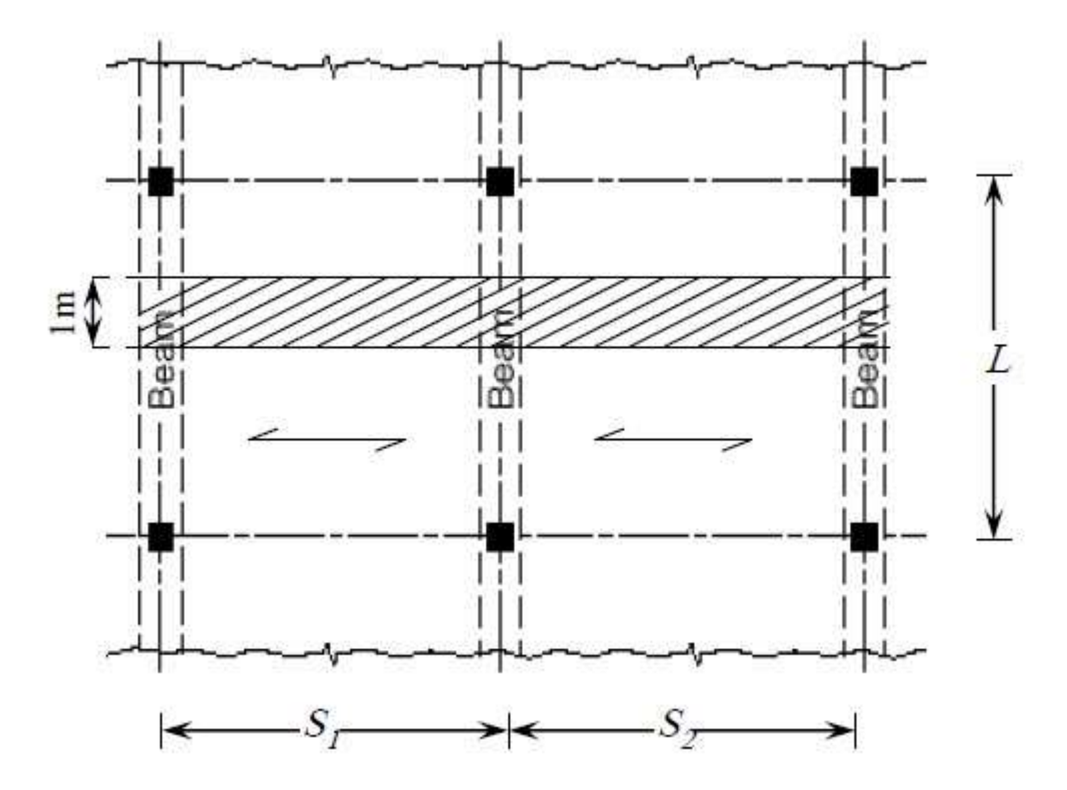
-Their own weight
-Weights of partitions applied directly on them
-Floor loads.



Design of one way SOLID slabs

One-way solid slabs are designed as a number of independent 1 m wide strips which span in the short direction and are supported on crossing beams. These strips are designed as rectangular beams.





$$\rho = \frac{1}{m}\left(1 - \sqrt{(1 - \frac{2mR}{fy})}\right)$$

$$m = \frac{fy}{0.85f'c}$$

$$R = \frac{M_u}{\varphi b d^2}$$

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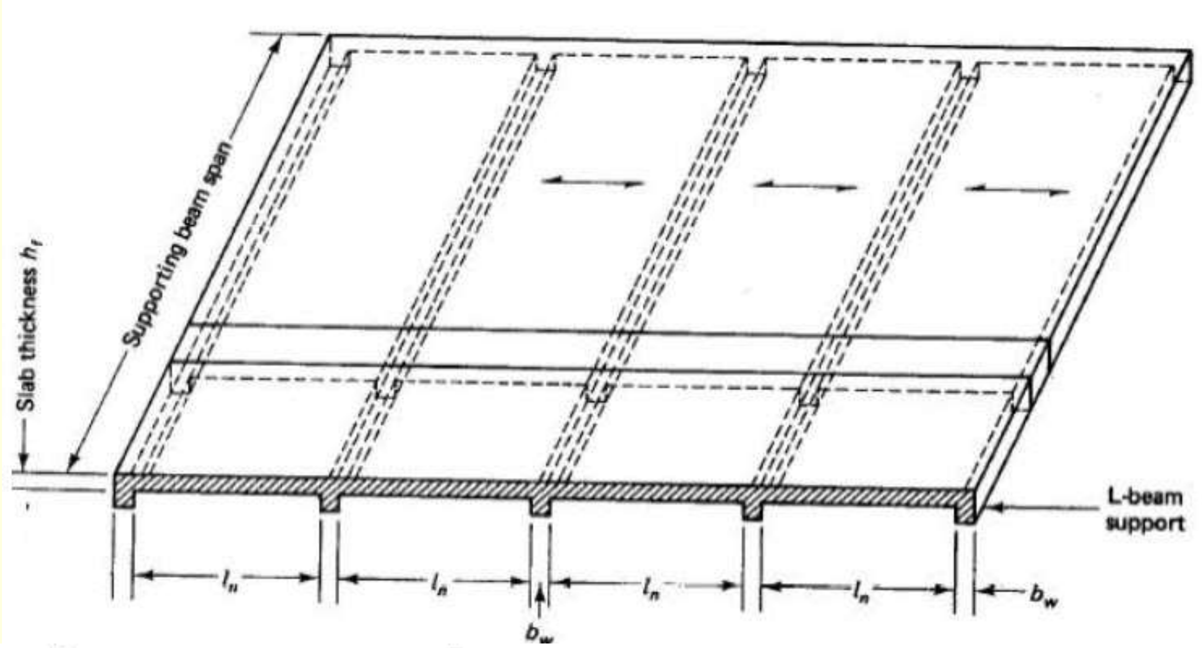
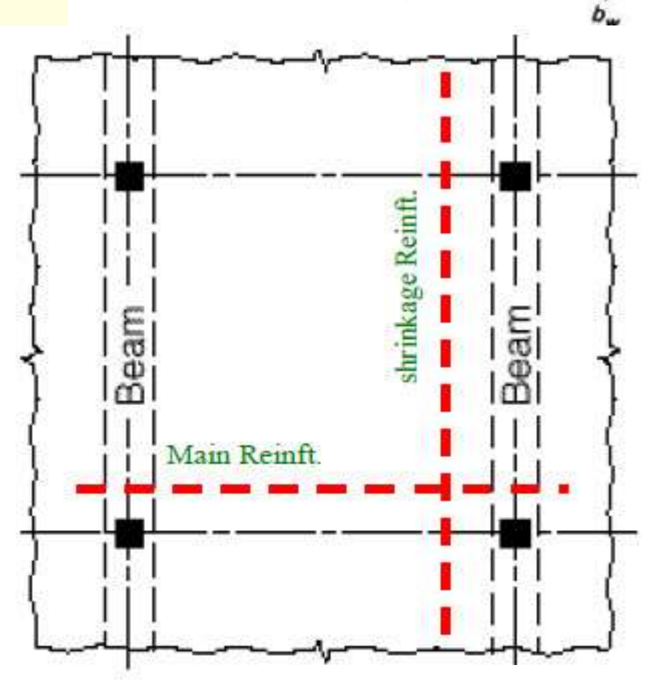
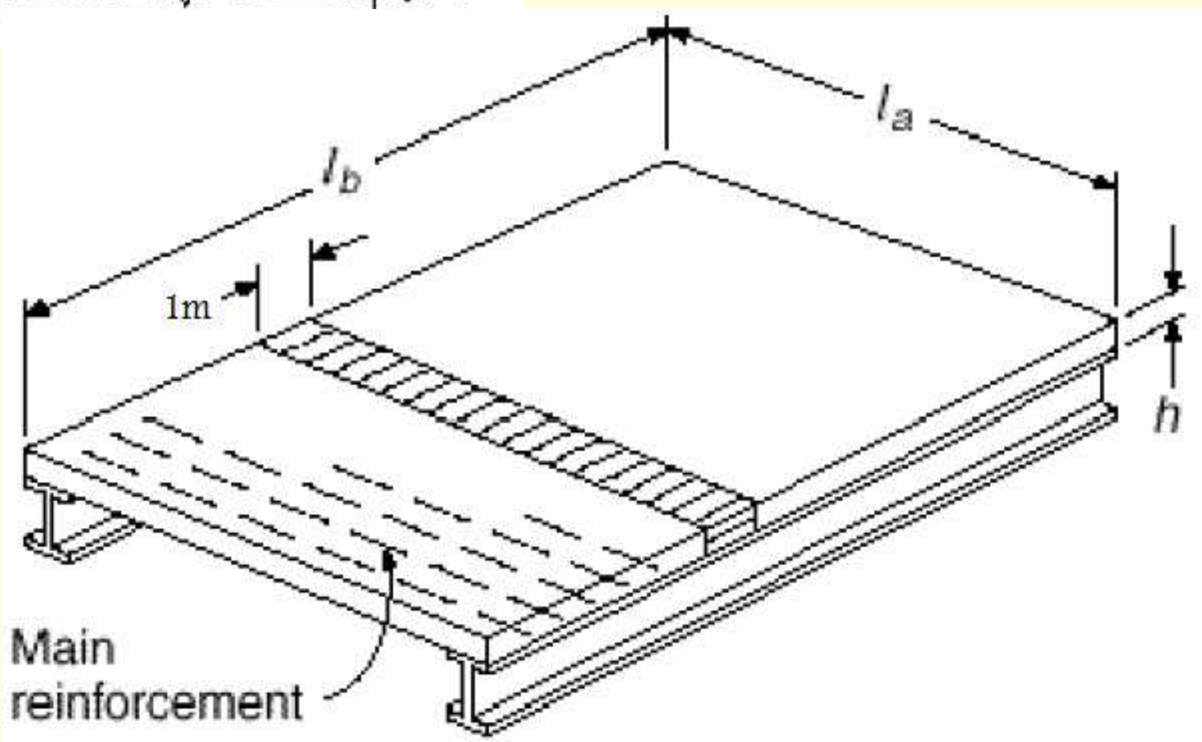






Check on tension/compression control (maximum allowed steel)

Method 1: Check ε_t

$$\varepsilon_t = \frac{d-c}{c}(0.003) \geq 0.005$$

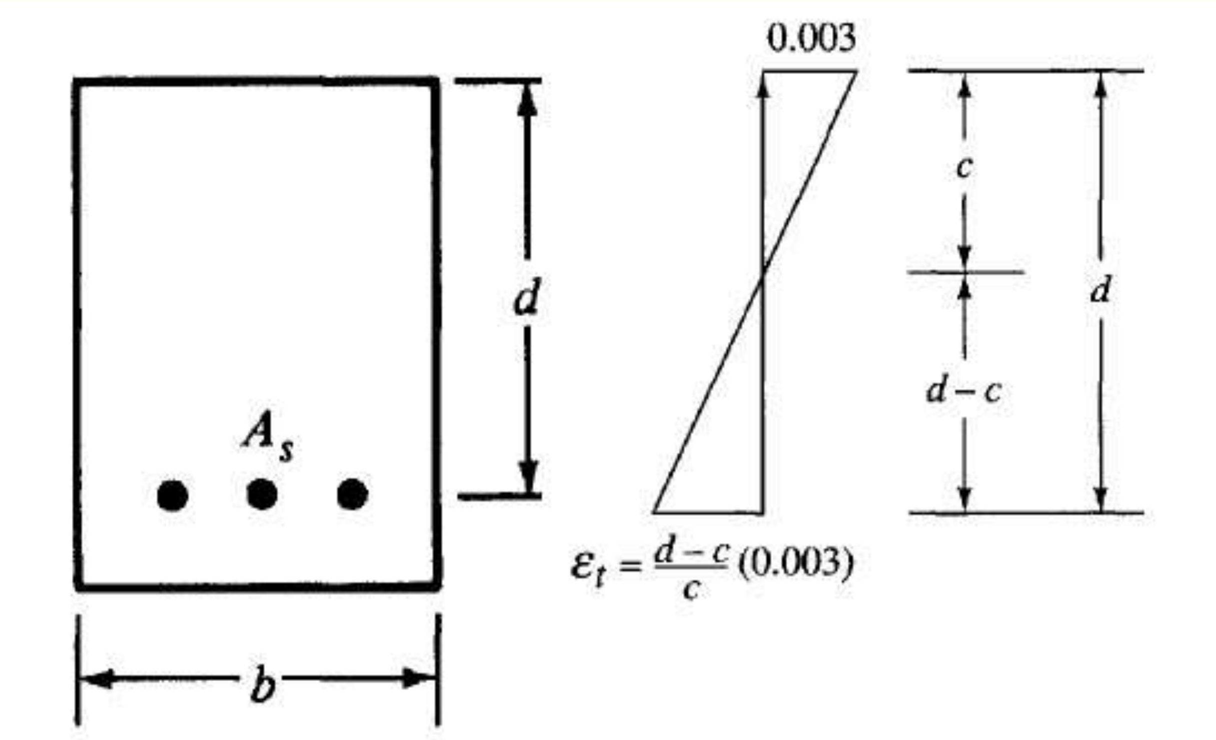
Method 2: Check ρ_{max}

$$C_{max} = \frac{0.003}{0.003+0.005}d$$

$$C_{max} = \frac{3}{8}d$$

$$a = \frac{A_s f_y}{0.85 f_c^{'}} = \frac{\rho f_y d}{0.85 f_c^{'}} = \beta_1 c = \beta_1 C_{max} = \frac{3}{8}d\beta_1$$

$$\rho_{max} = \frac{3}{8}\left(\frac{0.85\beta_1 f_c^{'}}{f_y}\right)$$



Shrinkage Reinforcement Ratio

According to **24.4.3.2** ACI Code 318M-14 and for $f_y = 420MPa$

$$\rho_{shrinkage} = 0.0018 \rightarrow A_{s,shrinkage} = 0.0018\, b * h$$

Where, **b** = width of strip, and **h** = slab thickness

Table 24.4.3.2—Minimum ratios of deformed shrinkage and temperature reinforcement area to gross concrete area

Reinforcement type	f_y, MPa	Minimum reinforcement ratio	
Deformed bars	< 420	0.0020	
Deformed bars or welded wire reinforcement	≥ 420	Greater of:	$\frac{0.0018 \times 420}{f_y}$
			0.0014

Minimum Reinforcement Ratio for Main Reinforcement

7.6.1.1 A minimum area of flexural reinforcement, $A_{s,min}$, shall be provided in accordance with Table 7.6.1.1.

Table 7.6.1.1—$A_{s,min}$ for nonprestressed one-way slabs

Reinforcement type	f_y, MPa	$A_{s,min}$	
Deformed bars	< 420	$0.0020A_g$	
Deformed bars or welded wire reinforcement	≥ 420	Greater of:	$\frac{0.0018 \times 420}{f_y} A_g$
			$0.0014A_g$

$$A_{s,min.} \geq A_{s,shrinkage} = 0.0018\, b * h$$

Check shear capacity of the section

$$V_u = \emptyset V_c = 0.17\emptyset \sqrt{f_c^{'}}\, b_w d$$

Otherwise enlarge depth of slab

Approximate Structural Analysis

ACI Code permits the use of the following approximate moments and shears for design of continuous beams and one-way slabs, provided:

• There are two or more spans.

• Spans are approximately equal, with the larger of two adjacent spans not greater than the shorter by more than 20 percent.

• Loads are uniformly distributed.

• Unfactored live load does not exceed three times the unfactored dead load.

• Members are of similar section dimensions along their lengths (prismatic).

M_u due to gravity loads shall be calculated in accordance with Table 6.5.2.

Table 6.5.2—Approximate moments for nonprestressed continuous beams and one-way slabs

Moment	Location	Condition	M_u
Positive	End span	Discontinuous end integral with support	$w_u \ell_n^2/14$
		Discontinuous end unrestrained	$w_u \ell_n^2/11$
	Interior spans	All	$w_u \ell_n^2/16$
Negative[1]	Interior face of exterior support	Member built integrally with supporting spandrel beam	$w_u \ell_n^2/24$
		Member built integrally with supporting column	$w_u \ell_n^2/16$
	Exterior face of first interior support	Two spans	$w_u \ell_n^2/9$
		More than two spans	$w_u \ell_n^2/10$
	Face of other supports	All	$w_u \ell_n^2/11$
	Face of all supports satisfying (a) or (b)	(a) slabs with spans not exceeding 3 m (b) beams where ratio of sum of column stiffnesses to beam stiffness exceeds 8 at each end of span	$w_u \ell_n^2/12$

[1]To calculate negative moments, ℓ_n shall be the average of the adjacent clear span lengths.

Vu **due to gravity loads shall be calculated in accordance with Table 6.5.4.**

Table 6.5.4—Approximate shears for nonprestressed continuous beams and one-way slabs

Location	V_u
Exterior face of first interior support	$1.15 w_u \ell_n / 2$
Face of all other supports	$w_u \ell_n / 2$

Summary of One-way Solid Slab Design Procedure

1- Select representative 1m wide design strip/strips to span in the short direction.

2- Choose a slab thickness to satisfy deflection control requirements. When several numbers of slab panels exist, select the largest calculated thickness.

3- Calculate the factored load W_u

4- Draw the shear force and bending moment diagrams for each of the strips.

5- Check adequacy of slab thickness in terms of resisting shear by satisfying the following equation: $V_u \leq 0.17\emptyset\sqrt{f_c'}b_w d$

Where b = 1000 mm. If the previous equation is not satisfied, go ahead and enlarge the thickness to do so.

6- Design flexural and shrinkage reinforcement.

Flexural reinforcement ratio is calculated from the following equation:-

$$\rho = \frac{1}{m}\left(1 - \sqrt{(1 - \frac{2mR}{fy})}\right)$$

$$m = \frac{fy}{0.85f'c}$$

$$R = \frac{M_u}{\varphi b d^2}$$

Where b = 1000 mm

You need to check tension controlled requirement, minimum reinforcement requirement and spacing of selected bars.

Compute the area of shrinkage reinforcement, where:

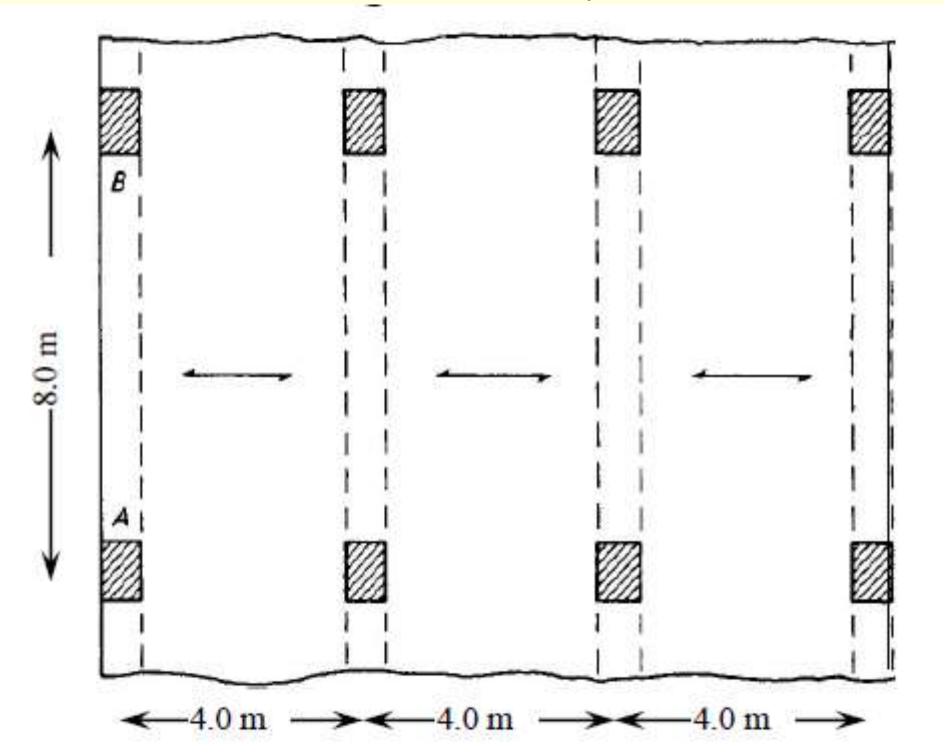
$A_{s,shrinkage} = 0.0018\, b * h$

Where b = 1000 mm.

7- Draw a plan of the slab and representative cross sections showing the dimensions and the selected reinforcement.

Example 1

Using the ACI-Code approximate structural analysis, design for a warehouse, a continuous one-way solid slab supported on beams 4.0 m apart as shown in Figure. Assume that the beam webs are 30 cm wide. The dead load is 3kN/m^2 in addition to the own weight of the slab, and the live load is 3kN/m^2. Use fc'=28MPa, fy= 420MPa



Solution:

1- Select a representative 1 m wide slab strip:

The selected representative strip is shown in the figure below

2- Select slab thickness:

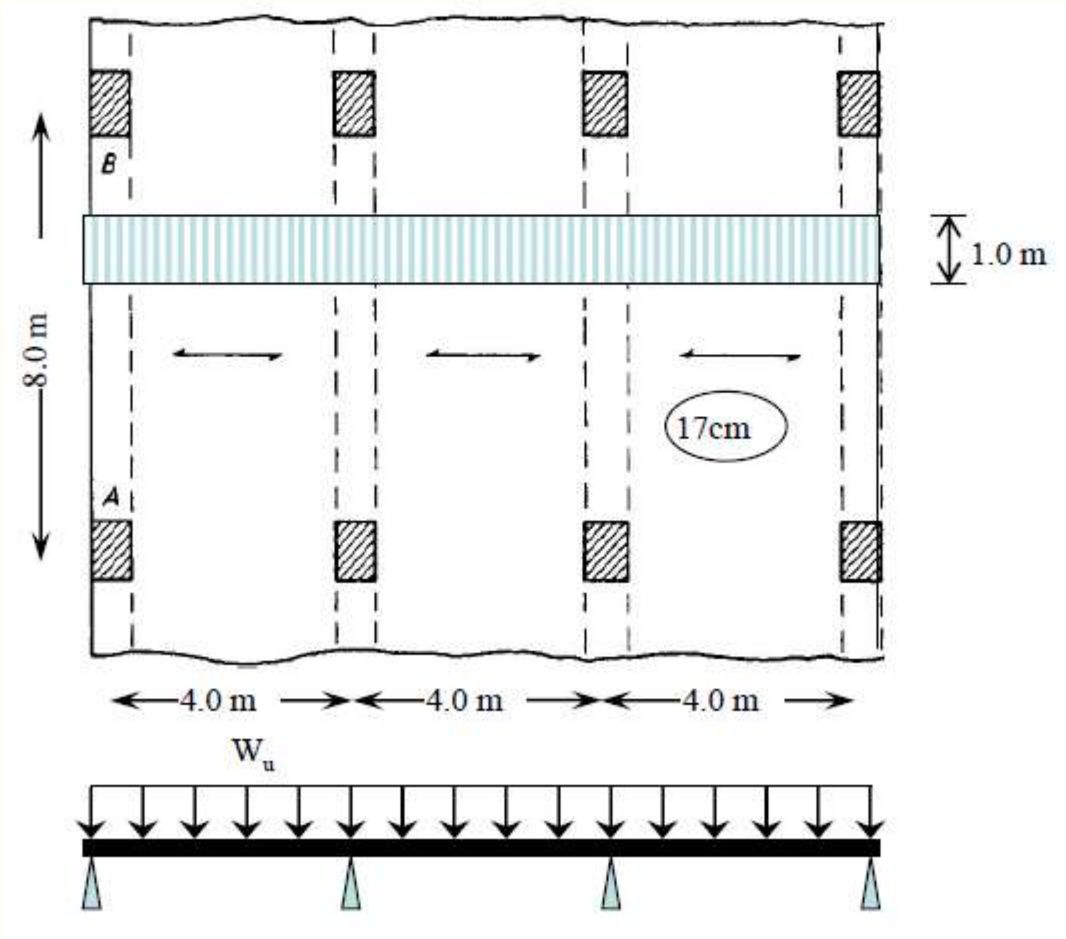
The clear span length, l_n

l_n = 4.0 – 0.30 = 3.70 m

For one-end continuous spans,

h_{min} = l/24 =4.0/24=0.167m

Slab thickness is taken as 17 cm



3- Calculate the factored load wu per unit length of the selected strip:

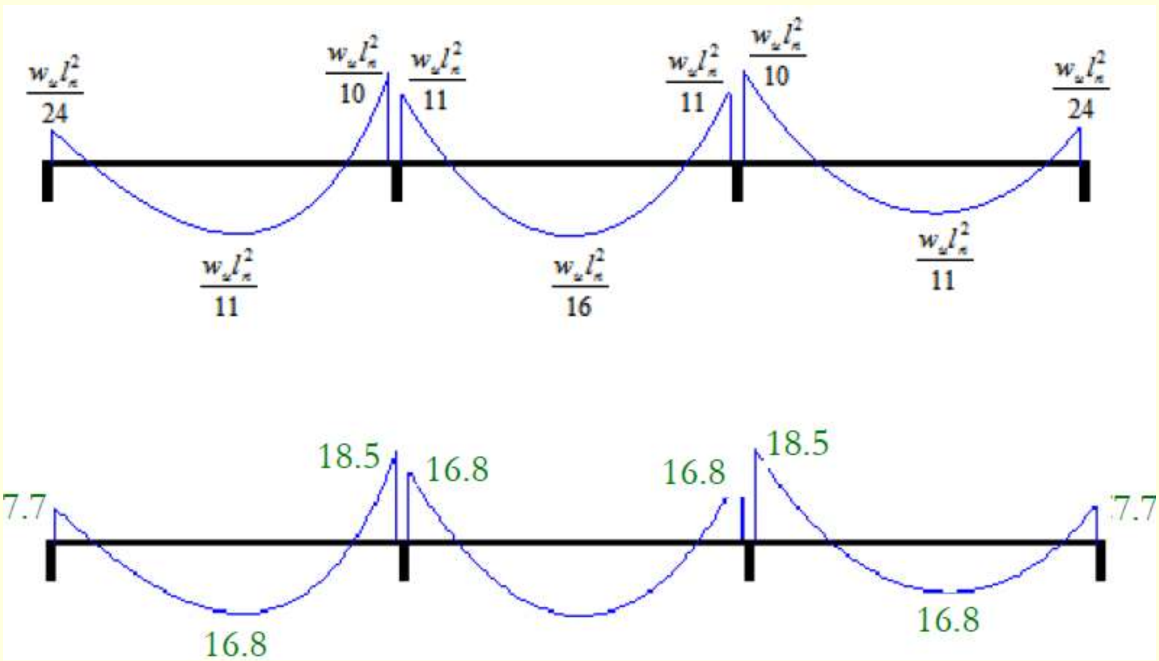
Own weight of slab = 0.17× 25 = 4.25 kN/m2

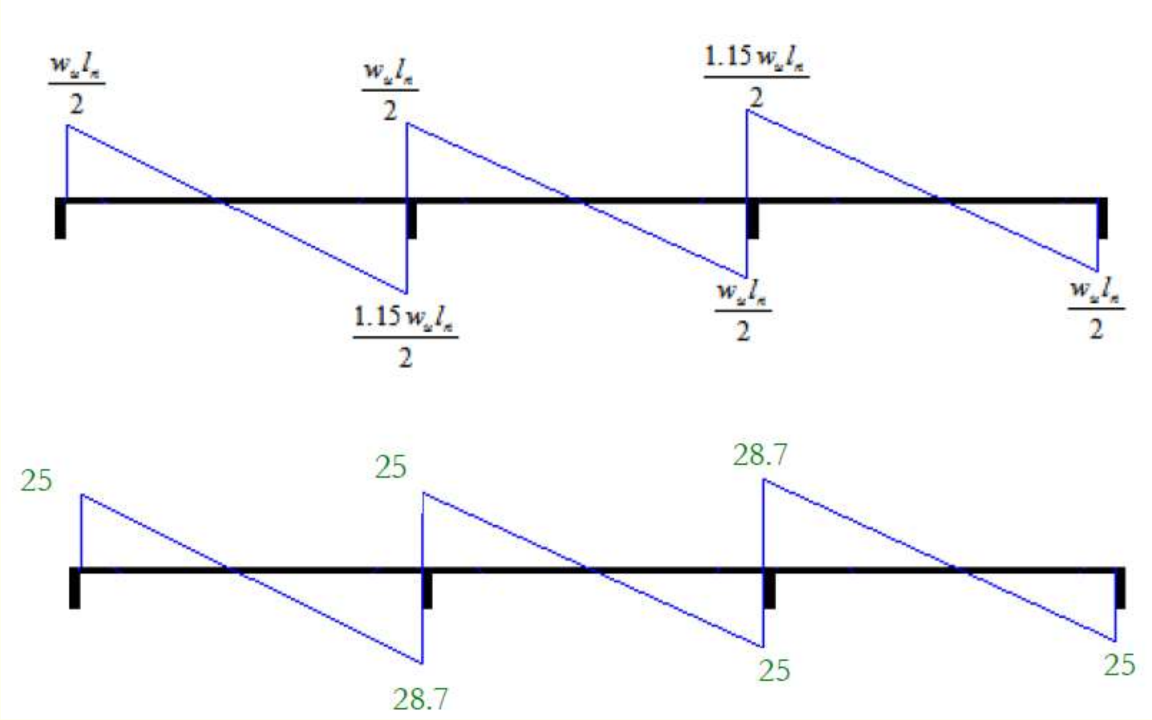
W_u= 1.20 (3+4.25) +1.60 (3)= 13.5 kN/m2

For a strip 1 m wide, W_u=13.5 kN/m

4- Evaluate the maximum factored shear forces and bending moments in the strip:

W_u=13.5 kN/m & l_n= 3.7m, Units of moment are in kN.m





Units of shear are in kN

5- Check slab thickness for beam shear:
Effective depth d = 170 – 20 –6 = 144 mm, assuming Φ12 mm bars.
Vu_{max} = 28.7 kN.

$$\emptyset V_c = 0.17\emptyset\sqrt{f_c^{'}}b_w d = 0.17(0.75)\sqrt{28} * 1000 * 144 * 10^{-3} = 95.8kN$$

i.e. Slab thickness is adequate in terms of resisting beam shear. No need to shear reinforcement.

6- Design flexural and shrinkage reinforcement:

Assume that Φ=0.9, For max negative moment, M_u = 18.5 kN.m

$$m = \frac{fy}{0.85f'c} = \frac{420}{0.85*28} = 17.647$$

$$R = \frac{M_u}{\varphi bd^2} = \frac{18.5 * 10^6}{0.9(1000)(144)^2} = 0.9913$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{(1 - \frac{2mR}{fy})}\right) = \frac{1}{17.647}\left(1 - \sqrt{(1 - \frac{2 * 17.647 * 0,9913}{420})}\right)$$
$$= 0.00241$$

$$A_{s)-ve} = \rho bd = 0.00241 * 1000 * 144 = 347\ mm^2$$

$$a = \frac{A_s f_y}{0.85 f_c^{'} b_w} = \frac{347 * 420}{0.85 * 28 * 1000} = 6.123\ mm$$

$$c = \frac{a}{\beta_1} = \frac{6.123}{0.85} = 7.2\ mm$$

$$\varepsilon_t = \frac{d-c}{c}(0{,}003) = \frac{144-7.2}{7.2}(0.003) = 0.057 > 0.005$$

Tension failure and $\emptyset = 0.9$

$$A_{s)min.} = 0.0018bh = 0.0018 * 1000 * 170 = 306mm^2 < A_{s)-ve}\ OK.$$

$$\frac{347}{1000} = \frac{79_{\emptyset 10}}{S} \longrightarrow S = 227.5\ mm$$

$$S_{max} = min.of\ (450mm\ or\ 3 * 170) = 450mm$$

$$use\ \varphi 10@\ 200mm$$

For max positive moment, M_u = 16.8 kN.m

$$m = \frac{fy}{0.85f'c} = \frac{420}{0.85*28} = 17.647$$

$$R = \frac{M_u}{\varphi bd^2} = \frac{16.8 * 10^6}{0.9(1000)(144)^2} = 0.9$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{(1 - \frac{2mR}{fy})}\right) = \frac{1}{17.647}\left(1 - \sqrt{(1 - \frac{2 * 17.647 * 0{,}9}{420})}\right)$$
$$= 000219$$

$$A_{s)+ve} = \rho bd = 0.00219 * 1000 * 144 = 315\ mm^2$$

$$a = \frac{A_s f_y}{0.85 f_c^{'} b_w} = \frac{315 * 420}{0.85 * 28 * 1000} = 5.56\ mm$$

$$c = \frac{a}{\beta_1} = \frac{5.56}{0.85} = 6.54\ mm$$

$$\varepsilon_t = \frac{d-c}{c}(0{,}003) = \frac{144-6.54}{6.54}(0.003) = 0.063 > 0.005$$

Tension failure and $\emptyset = 0.9$

$A_{s)min.} = 0.0018bh = 0.0018 * 1000 * 170 = 306mm^2 < A_{s)-ve}\ OK.$

$\frac{315}{1000} = \frac{79_{\emptyset 10}}{S} \longrightarrow S = 250\ mm$

$S_{max} = min.of\ (450mm\ or\ 3 * 170) = 450mm$

$use\ \varphi 10@\ 250mm$

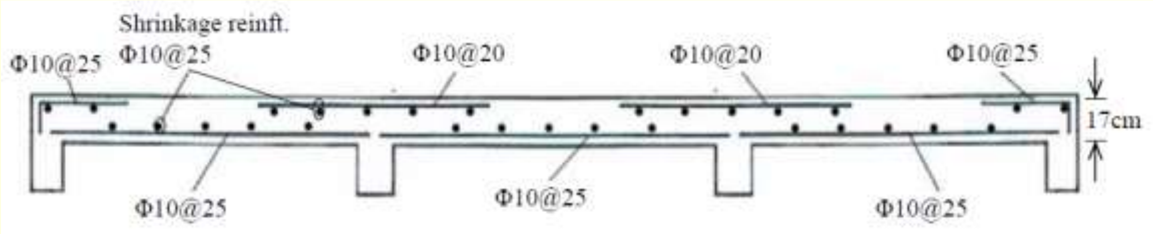
Calculate the area of shrinkage reinforcement:

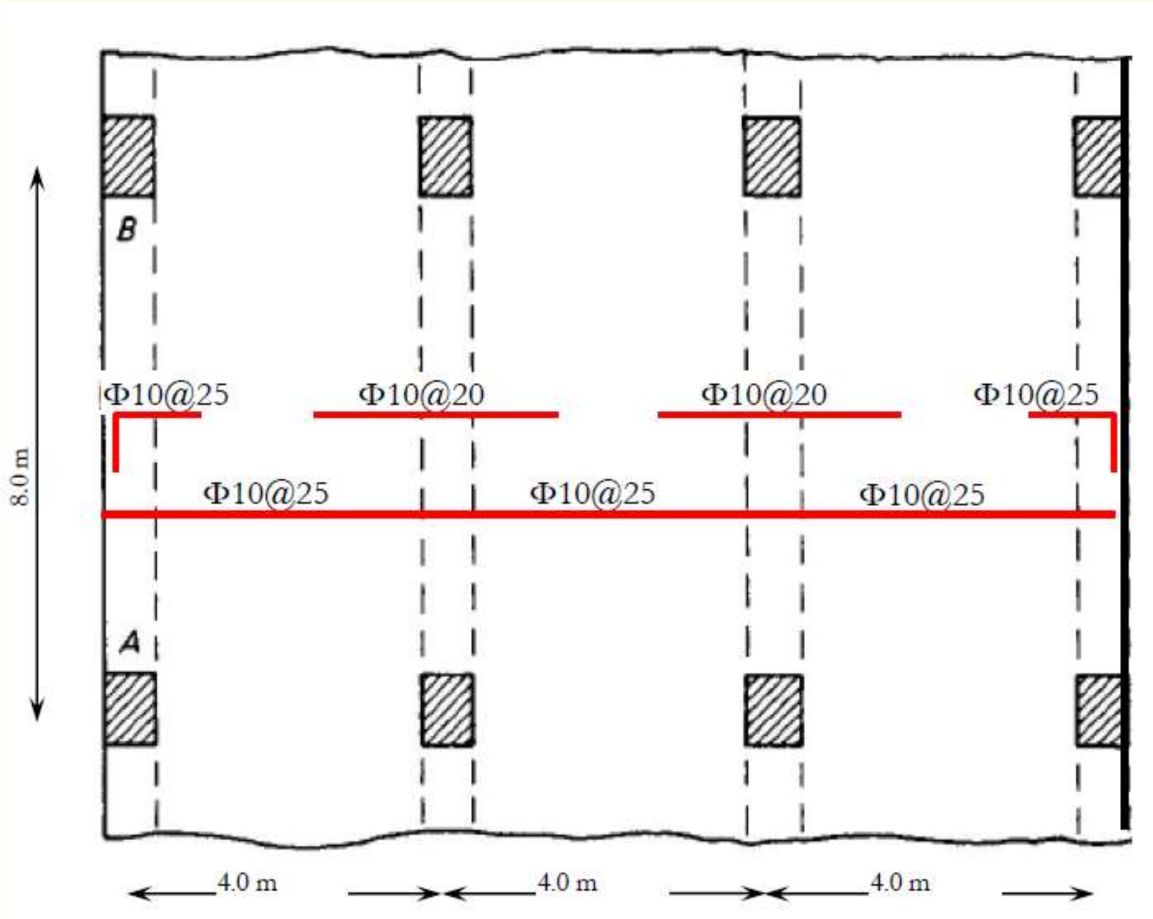
Area of shrinkage reinforcement = 0.0018 (1000) (170) = 306 mm^2/m

$\frac{306}{1000} = \frac{79_{\emptyset 10}}{S}\ S = 258\ mm$

$S_{max} \leq the\ smaller\ of\ \frac{5h_s}{450mm}$

For shrinkage reinforcement use Φ 10 mm @ 250 mm





Example 2:- A one-way single-span reinforced concrete slab has a simple span of 3m and carries a live load of 5.75 kN/m² and a dead load of 0.96 kN/m² in addition to its self-weight. Design the slab and the size and spacing of the reinforcement at mid-span assuming a simply supported moment, given: $f_{c`}=28MPa$ and $f_y=420MPa$

Solution

For simply supported span,

h_{min} = l/20 =3.0/20=0.15m

Slab thickness is taken as 150 mm.

Own weight of slab = 0.15× 25 = 3.6 kN/m²

W_u= 1.2(0.96+3.6) +1.6(5.75) = 14.7 kN/m²
For a strip 1 m wide, W_u=14.7 kN/m.
The clear span length, l_n

l_n = 3.0 m

W_u=14.7 kN/m & l_n= 3.0m.

Check slab thickness for beam shear:
Effective depth d = 150 – 20 –6 = 124 mm, assuming Φ12 mm bars.
Vu_{max} =$\frac{wl}{2}$ = 22.05 kN.

$$\emptyset V_c = 0.17\emptyset\sqrt{f_c^{'}}b_w d = 0.17(0.75)\sqrt{28} * 1000 * 124 * 10^{-3} = 83.66kN$$

i.e. Slab thickness is adequate in terms of resisting beam shear. No need to shear reinforcement.

Design flexural and shrinkage reinforcement:

Assume that Φ=0.9, for max positive moment,

M_u = = $\frac{wl^2}{8}$ =16.53 **kN.m**

$$m = \frac{fy}{0.85f'c} = \frac{420}{0.85*28} = 17.647$$

$$R = \frac{M_u}{\varphi bd^2} = \frac{16.53*10^6}{0.9(1000)(124)^2} = 1.195$$

$$\rho = \frac{1}{m}\left(1-\sqrt{(1-\frac{2mR}{fy})}\right) = \frac{1}{17.647}\left(1-\sqrt{(1-\frac{2*17.647*1.195}{420})}\right)$$
$$= 000292$$

$$A_{s)+ve} = \rho bd = 0.00292*1000*124 = 363\,mm^2/m$$

$$a = \frac{A_s f_y}{0.85 f_c^{'} b_w} = \frac{363*420}{0.85*28*1000} = 6.4\,mm$$

$$c = \frac{a}{\beta_1} = \frac{6.4}{0.85} = 7.53\,mm$$

$$\varepsilon_t = \frac{d-c}{c}(0{,}003) = \frac{124-6.54}{6.54}(0.003) = 0.046 > 0.005$$

Tension failure and $\emptyset = 0.9$

$$A_{s)min.} = 0.0018bh = 0.0018*1000*150 = 270mm^2/m < A_{s)+ve}\ OK.$$

$\frac{363}{1000} = \frac{113_{\emptyset 12}}{S} \Rightarrow S = 300\,mm$

$$S_{max} = min.of\ (450mm\ or\ 3*170) = 450mm$$

$use\ \varphi 12@\ 300mm$

Calculate the area of shrinkage reinforcement:

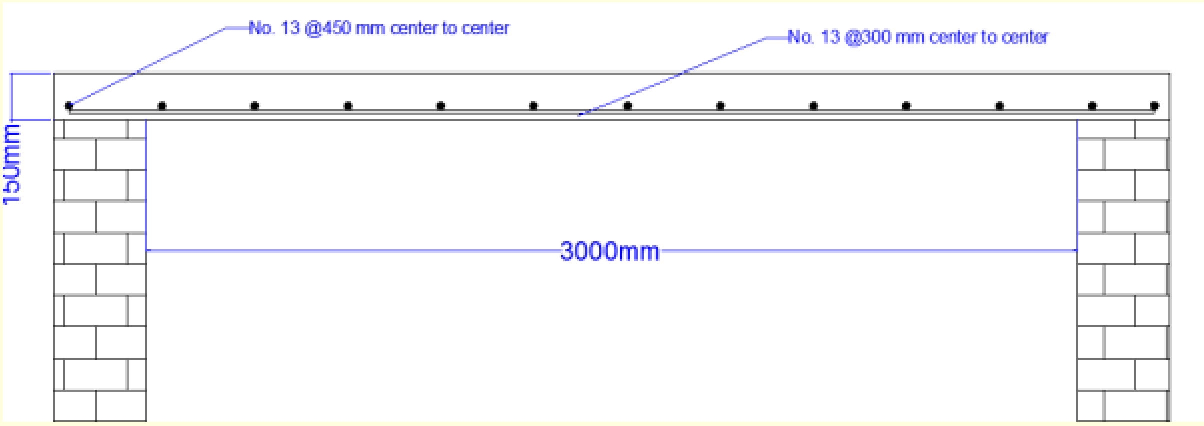
Area of shrinkage reinforcement = 0.0018 (1000) (150) = 270 mm^2/m

$\frac{270}{1000} = \frac{79_{\emptyset 10}}{S}\ S = 290\,mm$

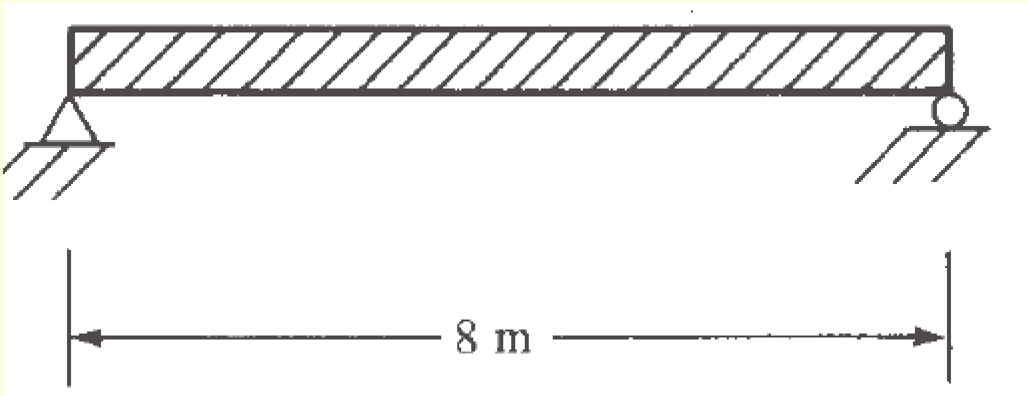
$$S_{max} \leq the\ smaller\ of\ \frac{5h_s}{450mm}$$

$$S_{max} = min.of\ (450mm)\ or(\ 5*150) = 750mm$$

For shrinkage reinforcement use Φ 10 mm @ 290 mm.



Example 3:- Design the one-way slab shown in the accompanying figure to support a live load of 12 kN/m^2.Do not use the ACI thickness limitation for deflections. Use $\boldsymbol{\rho=\rho_m}$, $` =28MPa$ and $f_y=420MPa$



Solution

Assume h slab=200 mm, Assume that Φ=0.9.

Own weight of slab = 0.2× 24 = 4.8 kN/m^2

W_u= 1.2(4.8) +1.6(12) = 24.96 kN/m^2

M_u = $= \frac{Wl^2}{8} = \frac{24.96*8^2}{8}$ =199.68 **kN.m**

$\boldsymbol{\rho_{max}=\frac{3}{8}\left(\frac{0.85\beta_1 f`_c}{f_y}\right)=0.018}$

$$bd^2 = \frac{M_u}{\varphi\rho f_y\left(1-0.59\rho\frac{f_y}{f`_c}\right)} = \frac{199.68*10^6}{0.9(0.018)420(1-0.59*0.018\frac{420}{28}}$$

$$= 34908341.48\, mm^3$$

$$\mathrm{d}^2 = 34908.34148$$
$$\mathrm{d} = 186.8\ \mathrm{mm}$$
$$\mathrm{h} = 186.6 + 20 + \frac{20}{2} = 216.6\ \mathrm{mm}$$

Own weight of slab = 0.2166 × 24 = 5.2 kN/m²

W_u= 1.2(5.2) +1.6(12) = 25.44 kN/m²

M_u **= =** $\frac{Wl^2}{8} = \frac{25.44*8^2}{8}$ **=203.5 kN.m**

$$\rho_{max=}\frac{3}{8}\left(\frac{0.85\beta_1 f'_c}{f_y}\right)=0.18$$

$$bd^2 = \frac{M_u}{\varphi\rho f_y\left(1-0.59\rho\frac{f_y}{f'_c}\right)} = \frac{203.5*10^6}{0.9(0.018)420(1-0.59*0.018\frac{420}{28})} = 35576159.31 mm^3$$

$$\mathrm{d}^2 = 35576.15931$$
$$\mathrm{d} = 188.6\ \mathrm{mm}$$
$$\mathrm{h} = 188.6 + 20 + \frac{20}{2} = 218.6\ \mathrm{mm} \approx 220\ \mathrm{mm}$$

Check slab thickness for beam shear:

Effective depth d = 220 – 20 –10 = 190 mm, assuming Φ20 mm bars.

Vu_{max} =$\frac{wl}{2} = \frac{25.44*8}{2}$ = 101.76 kN.

$$\emptyset V_c = 0.17\emptyset\sqrt{f'_c}b_w d = 0.17(0.75)\sqrt{28}*1000*190*10^{-3} = 128.18kN$$

i.e. Slab thickness is adequate in terms of resisting beam shear. No need to enlarge the section.

Design flexural and shrinkage reinforcement:

For max positive moment, middle span.

M_u **=** $203.5\ kN.m$

$$A_{s)+ve} = \rho bd = 0.018*1000*190 = 3420\ mm^2/m$$

$$a=\frac{A_s f_y}{0.85 f_c^{'} b_w}=\frac{3420*420}{0.85*28*1000}=60.35\ mm$$

$$c=\frac{a}{\beta_1}=\frac{60.35}{0.85}=71\ mm$$

$$\varepsilon_t=\frac{d-c}{c}(0{,}003)=\frac{190-71}{71}(0.003)=0.00502>0.005$$

Tension failure and $\emptyset=0.9$(note we used maximum ratio of steel)

$$A_{s)min.}=0.0018bh=0.0018*1000*220=396mm^2/m<A_{s)+ve}\ OK.$$

$$\frac{3420}{1000}=\frac{314_{\emptyset 20}}{S} \longrightarrow S=91.9\ mm$$

$$\frac{3420}{1000}=\frac{490_{\emptyset 25}}{S} \qquad S=143\ mm$$

$$S_{max}=min.of\ (450mm\ or\ 3*220)=450mm$$

$$use\ \varphi 25@\ 140mm$$

Calculate the area of shrinkage reinforcement:

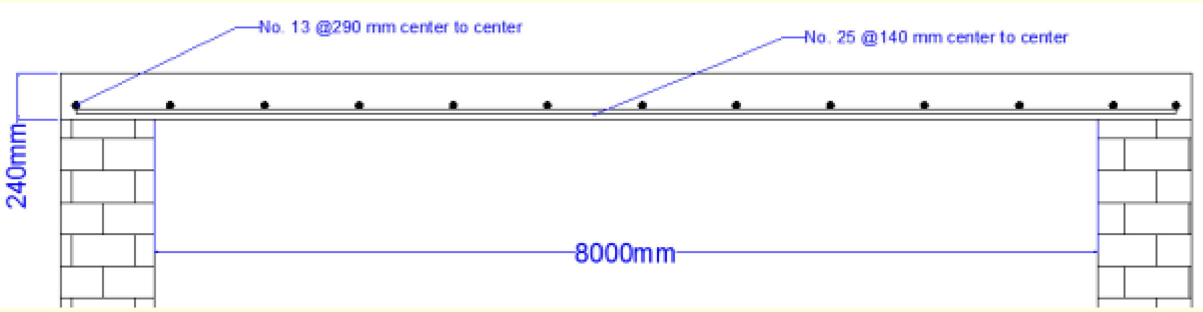
Area of shrinkage reinforcement = 0.0018 (1000) (220) = 396mm^2/m

$$\frac{396}{1000}=\frac{113_{\emptyset 12}}{S} \qquad S=285\ mm$$

$$S_{max}\leq the\ smaller\ of\ \frac{5h_s}{450mm}$$

$$S_{max}=min.of\ (450mm)\ or(\ 5*150)=750mm$$

For shrinkage reinforcement use Φ 12 mm @ 280 mm.



Deflections

GENERAL CONSIDERATIONS

The ACI code provisions for control of deflections are concerned only with deflections that occur at service load levels under static conditions and may not apply to loads with strong dynamic characteristics such as those due to earthquakes, transient winds, and vibration of machinery.

Note:

Two methods are given in the Code for controlling deflections:-

- For nonprestressed one-way slabs and beams, including composite members, the minimum overall thickness required by ACI-318M-14 7.3.1 and 9.3.1 is considered to satisfy the requirements of the Code for members not supporting or attached to nonstructural elements likely to be damaged by large deflections.

7.3.1 *Minimum slab thickness*

7.3.1.1 For solid nonprestressed slabs not supporting or attached to partitions or other construction likely to be damaged by large deflections, overall slab thickness ***h*** shall not be less than the limits in Table 7.3.1.1, unless the calculated deflection limits of 7.3.2 are satisfied.

Table 7.3.1.1—Minimum thickness of solid nonprestressed one-way slabs

Support condition	Minimum h[1]
Simply supported	$\ell/20$
One end continuous	$\ell/24$
Both ends continuous	$\ell/28$
Cantilever	$\ell/10$

[1]Expression applicable for normalweight concrete and f_y = 420 MPa. For other cases, minimum h shall be modified in accordance with 7.3.1.1.1 through 7.3.1.1.3, as appropriate.

9.3.1 *Minimum beam depth*

9.3.1.1 For nonprestressed beams not supporting or attached to partitions or other construction likely to be damaged by large deflections, overall beam depth ***h*** shall satisfy the limits in Table 9.3.1.1, unless the calculated deflection limits of 9.3.2 are satisfied.

Table 9.3.1.1—Minimum depth of nonprestressed beams

Support condition	Minimum h[1]
Simply supported	$\ell/16$
One end continuous	$\ell/18.5$
Both ends continuous	$\ell/21$
Cantilever	$\ell/8$

[1]Expressions applicable for normalweight concrete and Grade 420 reinforcement. For other cases, minimum h shall be modified in accordance with 9.3.1.1.1 through 9.3.1.1.3, as appropriate.

- For nonprestressed members that do not meet these minimum thickness requirements, for nonprestressed one-way members that support or are attached to nonstructural elements likely to be damaged by large deflections, and for prestressed flexural members, deflections are required to be calculated by 24.2.3 through 24.2.5. Calculated deflections are limited to the values in Table 24.2.2.

Table 24.2.2—Maximum permissible calculated deflections

Member	Condition		Deflection to be considered	Deflection limitation
Flat roofs	Not supporting or attached to nonstructural elements likely to be damaged by large deflections		Immediate deflection due to maximum of L_r, S, and R	$\ell/180$[1]
Floors			Immediate deflection due to L	$\ell/360$
Roof or floors	Supporting or attached to nonstructural elements	Likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements, which is the sum of the time-dependent deflection due to all sustained loads and the immediate deflection due to any additional live load[2]	$\ell/480$[3]
		Not likely to be damaged by large deflections		$\ell/240$[4]

[1]Limit not intended to safeguard against ponding. Ponding shall be checked by calculations of deflection, including added deflections due to ponded water, and considering time-dependent effects of sustained loads, camber, construction tolerances, and reliability of provisions for drainage.

[2]Time-dependent deflection shall be calculated in accordance with 24.2.4, but shall be permitted to be reduced by amount of deflection calculated to occur before attachment of nonstructural elements. This amount shall be calculated on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered.

[3]Limit shall be permitted to be exceeded if measures are taken to prevent damage to supported or attached elements.

[4]Limit shall not exceed tolerance provided for nonstructural elements.

1-Immediate Deflection of Beams and One-Way Slabs (Nonprestressed)

Initial or short-term deflections of beams and one-way slabs occur immediately on the application of load to a structural member. The principal factors that affect the immediate deflection of a member are:
a. magnitude and distribution of load,
b. span and restraint condition,
c. section properties and steel percentage,
d. material properties.
e. amount and extent of flexural cracking.

2-Long-Term Deflection of Beams and One-Way Slabs (Nonprestressed)

Beams and one-way slabs subjected to sustained loads experience long-term deflections. These deflections may be two to three times as large as the immediate elastic deflection that occurs when the sustained load is applied. The long-term deflection is caused by the effects of shrinkage and creep, the formation of new cracks and the widening of earlier cracks. The principal factors that affect long-term deflections are:
a. stresses in concrete
b. amount of tensile and compressive reinforcement
c. member size
d. curing conditions
e. temperature
f. relative humidity
g. age of concrete at the time of loading
h. duration of loading.

Note:

Total deflection = Immediate Deflection + Long-Term Deflection

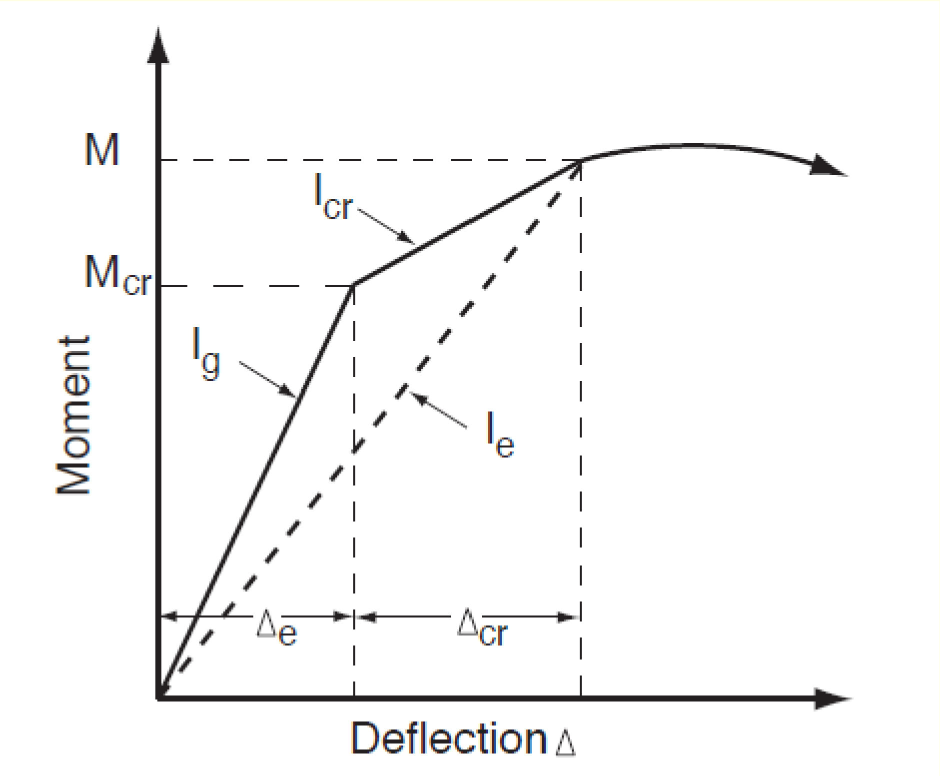
Calculation of immediate deflections

The idealized short-term deflection of a typical reinforced concrete beam is shown in Fig. below. There are two distinct phases of behavior:

(i) Uncracked behavior, when the applied moment (M_a) is less than the cracking moment (M_{cr}).

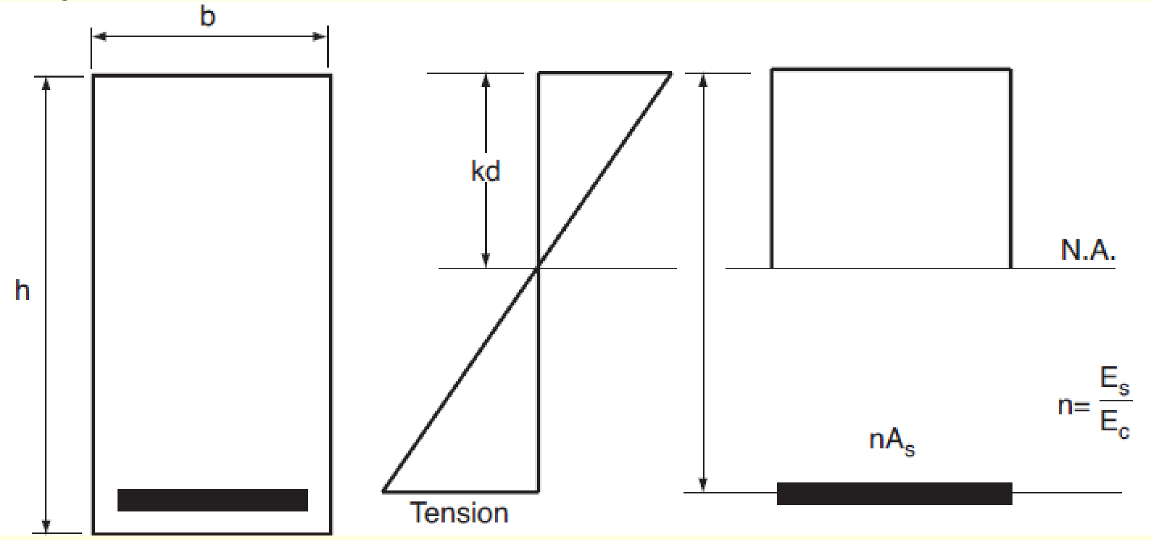
(ii) Cracked behavior, when the applied moment (M_a) is greater than the cracking moment (M_{cr}).

Two different values for the moment of inertia would therefore be used for calculating the deflections: the gross moment of inertia (I_g) for the uncracked section, and the reduced moment of inertia for the cracked section (I_{cr}).



Note

For the uncracked rectangular beam shown in Fig. above, the gross moment of inertia is used ($I_g = bh^3/12$). The moment of inertia of a cracked beam with tension reinforcement (I_{cr}) is computed in the following manner:



Taking moment of areas about the neutral axis

$$b \times kd \times \frac{kd}{2} = nA_s(d - kd)$$

Find "kd"

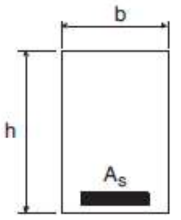
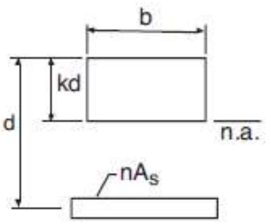
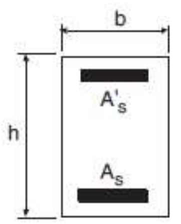
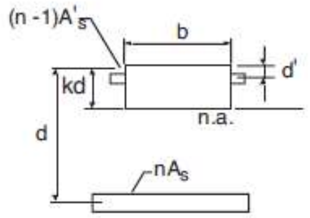
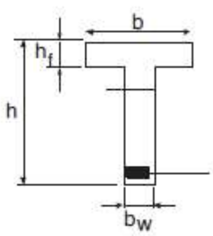
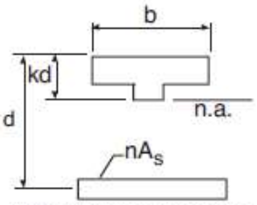
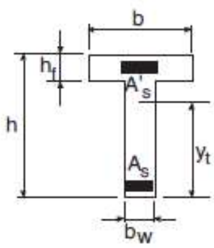
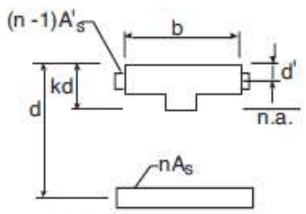
Use $$B = \frac{b}{nA_s}$$

$$kd = \frac{\sqrt{2Bd + 1} - 1}{B}$$

Moment of inertia of cracked section about neutral axis,

$$I_{cr} = \frac{b(kd)^3}{3} + nA_s(d - kd)^2$$

Expressions for computing the cracked moment of inertia for sections with compression reinforcement and flanged sections, which are determined in a similar manner, are given in Table below.

Gross Section	Cracked Transformed Section	Gross and Cracked Moment of Inertia
	 Without compression steel	$n = \frac{E_s}{E_c}$ $B = \frac{b}{(nA_s)}$ $I_g = \frac{bh^3}{12}$ Without compression steel $kd = \left(\sqrt{2dB+1}-1\right)/B$ $I_{cr} = b(kd)^3/3 + nA_s(d-kd)^2$
	 With compression steel	With compression steel $r = (n-1)A'_s/(nA_s)$ $kd = \left[\sqrt{2dB(1+rd'/d)+(1+r)^2}-(1+r)\right]/B$ $I_{cr} = b(kd)^3/3 + nA_s(d-kd)^2 + (n-1)A'_s(kd-d')^2$
	 Without compression steel	$n = \frac{E_s}{E_c}$ $C = b_w/(nA_s)$, $f = h_f(b-b_w)/(nA_s)$, $y_t = h - 1/2\left[(b-b_w)h_f^2 + b_w h^2\right]/\left[(b-b_w)h_f + b_w h\right]$ $I_g = (b-b_w)h_f^3/12 + b_w h^3/12 + (b-b_w)h_f(h-h_f/2-y_t)^2 +$ $b_w h(y_t - h/2)^2$ Without compression steel $kd = \left[\sqrt{C(2d+h_f f)+(1+f)^2}-(1+f)\right]/C$ $I_{cr} = (b-b_w)h_f^3/12 + b_w(kd)^3/3 + (b-b_w)h_f(kd-h_f/2)^2 +$ $nA_s(d-kd)^2$
	 With compression steel	With compression steel $kd = \left[\sqrt{C(2d+h_f f+2rd')+(f+r+1)^2}-(f+r+1)\right]/C$ $I_{cr} = (b-b_w)h_f^3/12 + b_w(kd)^3/3 + (b-b_w)h_f(kd-h_f/2)^2 +$ $nA_s(d-kd)^2 + (n-1)A'_s(kd-d')^2$

Effective Moment of Inertia for Beams and One-Way Slabs (Nonprestressed)

The flexural rigidity EI of a beam may not be constant along its length because of varying amounts of steel and cracking at different sections along the beam. This, and other material related sources of variability, makes the exact prediction of deflection difficult in practice.

The effective moment of inertia of cantilevers, simple beams, and continuous beams between inflection points is given by:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \leq I_g \qquad (24.2.3.5a)$$

Where:

$$M_{cr} = \frac{f_r I_g}{y_t} \qquad (24.2.3.5a)$$

$$f_r = 0.62\sqrt{f_c'}$$

M_a = maximum service load moment (unfactored) at the stage for which deflections are being considered.

y_t= the maximum distance from the tension side to the neutral axis.

NOTE 24.2.3.6

For continuous one-way slabs and beams, $\boldsymbol{I_e}$ shall be permitted to be taken as the average of values obtained from Eq. (24.2.3.5a) for the critical positive and negative moment sections.

NOTE 24.2.3.7

For prismatic one-way slabs and beams, $\boldsymbol{I_e}$ shall be permitted to be taken as the value obtained from Eq. (24.2.3.5a) at mid-span for simple and continuous spans, and at the support for cantilevers.

The initial or short-term deflection (Δ_i) for cantilevers and simple and continuous beams may be computed using the following elastic equation given below. For continuous beams, the mid-span deflection may usually be used as an approximation of the maximum deflection.

$$\Delta_i = \frac{5kM_a l^2}{48E_c I_e} \qquad (\#1)$$

Where:

M_a = the support moment for cantilevers and the mid-span moment for simple and continuous beams.

l = the span length.

$$E_c = 4700\sqrt{f_c'} \qquad (19.2.3.1)$$

For uniformly distributed loading w, the theoretical values of the deflection coefficient K are shown in Table below. Since deflections are logically computed for a given continuous span based on the same loading pattern as for maximum positive moment, Eq. (#1) is thought to be the most convenient form for a deflection equation.

		K
1.	Cantilevers (deflection due to rotation at supports not included)	2.40
2.	Simple beams	1.0
3.	Continuous beams	1.2-0.2 M_o/M_a
4.	Fixed-hinged beams (midspan deflection)	0.80
5.	Fixed-hinged beams (maximum deflection using maximum moment)	0.74
6.	Fixed-fixed beams	0.60

M_o = Simple span moment at midspan $\left(\frac{w\ell^2}{8}\right)$

M_a = Net midspan moment.

<u>Calculation of long-term deflections</u> *(time-dependent deflections)*

According to **24.2.4.1.1**, additional long-term deflections due to the combined effects of shrinkage and creep from sustained loads $\Delta_{(cp+sh)}$ may be estimated by multiplying the immediate deflection caused by the sustained load $(\Delta i)_{sus}$ by the factor λ_Δ ; i.e.

$$\frac{\Delta_i}{\Delta_{i_{sus}}} = \frac{W_{total}}{w_{dead} + W_{live} * \%\, sustained\ live\ load}$$

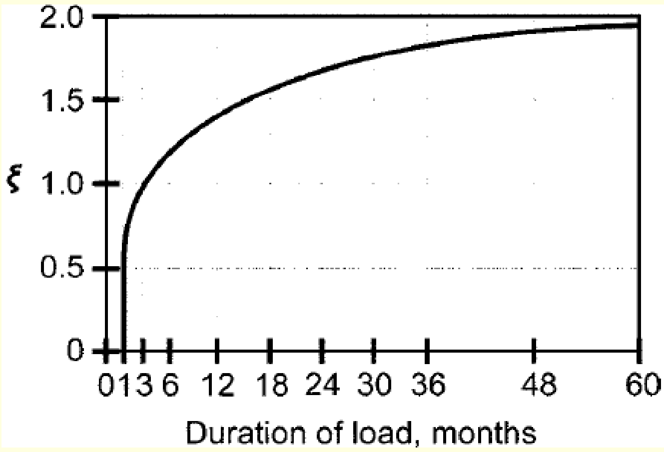
$\Delta_{(cp+sh)}$ =long term deflection= $\lambda(\Delta i)_{sus}$

$$\lambda = \frac{\xi}{1 + 50\rho'}$$

Values for ξ are given in Table 24.2.4.1.3 for different durations of sustained load. Figure R24.2.4.1in the commentary to the code shows the variation of ξ for periods up to 5 years. The compression steel ($\rho' = A_s' / bd$) is computed at the support section for cantilevers and the mid-span section for simple and continuous spans. Note that sustained loads include dead load and that portion of live load that is sustained.

Table 24.2.4.1.3—Time-dependent factor for sustained loads

Sustained load duration, months	Time-dependent factor ξ
3	1.0
6	1.2
12	1.4
60 or more	2.0



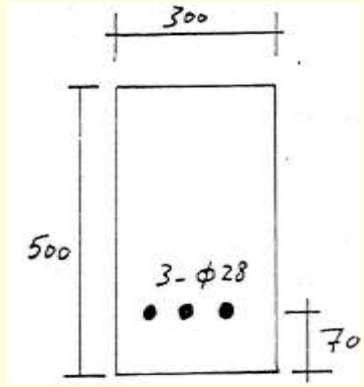
<u>Deflection Limits</u>

Deflections computed using the preceding methods are compared to the limits given in Table 24.2.2. The commentary gives information for the correct application of these limits, including consideration of deflections occurring prior to installation of partitions.

Dr. Majid albana

Example

A simply supported beam with span of 6 m carry a dead load of 15kN/m and live load of 10kN/m. Find the total maximum deflection, consider that 30% of live load are sustained. $f'_c = 20.7MPa$



$$W = W_D + W_l = 15 + 10 = \frac{25kN}{m}$$

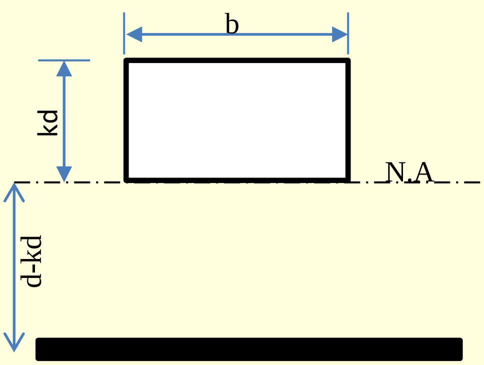
$$M_a = \frac{Wl^2}{8} = \frac{25 \times 6^2}{8} = 112.5kN.m$$

$$I_g = \frac{bh^3}{12} = \frac{300(500)^3}{12} = 3.125 \times 10^9 mm^4$$

$$f_{cr} = \frac{M_{cr} \times 10^6 \times c_t}{I_g}$$

$$0.62\sqrt{20.7} = \frac{M_{cr} \times 10^6 \times 250}{3.125 \times 10^9} \quad \rightarrow \quad M_{cr} = 35.26kN.m$$

$$M_a > M_{cr}\ calculate\ I_e = (\frac{M_{cr}}{M_a})^3 I_g + \left[1 - (\frac{M_{cr}}{M_a})^3\right] I_{cr} \le I_g$$



$$n = \frac{E_S}{E_c} = \frac{200000}{4700\sqrt{20.7}} \approx 9$$

$$A_s = 3 \times 28^2 \frac{\pi}{4} = 1847\ mm^2$$

$$B = \frac{b}{nA_s} = \frac{300}{9 \times 1847} = 0.01804$$

$$kd = \frac{\sqrt{2Bd + 1} - 1}{B} = \frac{\sqrt{2 \times 0.01804 \times 430 + 1} - 1}{0.01804} = 170mm$$

$$I_{cr} = \frac{b(kd)^3}{3} + nA_s(d - kd)^2 = \frac{300(170)^3}{3} + 9 \times 1847(430 - 170)^2 = 1.615 \times 10^9 mm^4$$

$$I_e = \left(\frac{35.26}{112.5}\right)^3 3.125 \times 10^9 + \left[1 - \left(\frac{35.26}{112.5}\right)^3\right] 1.615 \times 10^9 = 1.661 \times 10^9 mm^4$$

$$\Delta_i = \frac{5Wl^4}{384E_cI_e} = \frac{5 \times 25 \times 6000^4}{384 \times 4700\sqrt{20.7} \times 1.661 \times 10^9} = 11.87mm$$

To find long term deflection first we find $\Delta_{i_{sus}}$

$\Delta_i = 11.87mm \qquad 25kN$

$\Delta_{i_{sus}} = ?\,mm \qquad 15 + 10 \times \frac{30}{100}$

$\Delta_{i_{sus}} = 8.54mm$

$\xi = 2$

$$\lambda = \frac{\xi}{1 + 50\rho'} = \frac{2}{1 + (50)0} = 2$$

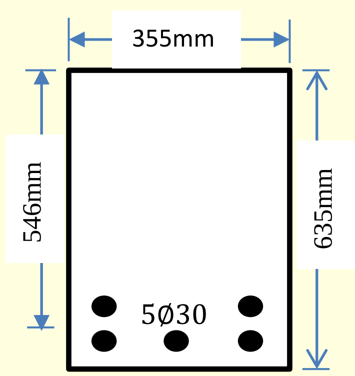
$\Delta_{(cp+sh)}$ =long term deflection=$\Delta_{sus} = \lambda \Delta_{i_{sus}} = 2 \times 8.54 = 17.09mm$

Total deflection = Immediate Deflection + Long-Term Deflection

$\Delta_{total} = \Delta_i + \Delta_{sus} = 11.87 + 17.09 = 28.96\,mm$

Example

A simply supported beam with span of 7.5 m carry a dead load of 29kN/m including self-weight and live load of 20.5kN/m. Find the total maximum deflection, consider $f'_c = 27.6MPa$.



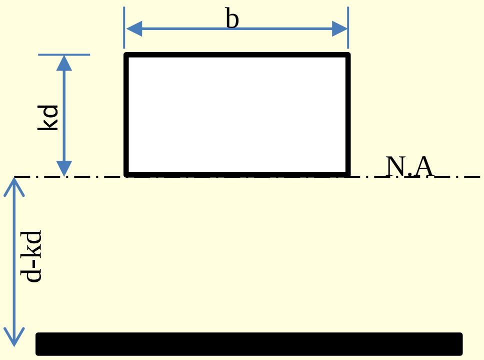
$$W = W_D + W_l = 29 + 20.5 = \frac{49.5kN}{m}$$

$$M_a = \frac{Wl^2}{8} = \frac{49.5 \times (7.5)^2}{8} = 348\,kN.m$$

$$I_g = \frac{bh^3}{12} = \frac{355(635)^3}{12} = 7.574 \times 10^9 mm^4$$

$$f_{cr} = \frac{M_{cr} \times 10^6 \times c_t}{I_g}$$

$$0.62\sqrt{27.6} = \frac{M_{cr} \times 10^6 \times 317.5}{7.574 \times 10^9} \quad \rightarrow \quad M_{cr} = 77.7kN.m$$



$M_a > M_{cr}$ $calculate$ $I_e = (\frac{M_{cr}}{M_a})^3 I_g + \left[1 - (\frac{M_{cr}}{M_a})^3\right] I_{cr} \leq I_g$

$$n = \frac{E_S}{E_c} = \frac{200000}{4700\sqrt{27.6}} \approx 8$$

$$A_s = 5 \times 30^2 \frac{\pi}{4} = 3534.3\ mm^2$$

$$B = \frac{b}{nA_s} = \frac{355}{8 \times 3534.3} = 0.012555$$

$$kd = \frac{\sqrt{2Bd+1} - 1}{B} = \frac{\sqrt{2 \times 0.012555 \times 546 + 1} - 1}{0.012555} = 225.83mm$$

$$I_{cr} = \frac{b(kd)^3}{3} + nA_s(d-kd)^2 = \frac{355(225.83)^3}{3} + 8 \times 3534.3(546 - 225.83)^2 = 4.2616 \times 10^9 mm^4$$

$$I_e = \left(\frac{77.7}{348}\right)^3 7.574 \times 10^9 + \left[1 - \left(\frac{77.7}{348}\right)^3\right] 4.2616 \times 10^9 = 4.3 \times 10^9 mm^4$$

$$\Delta_i = \frac{5Wl^4}{384E_cI_e} = \frac{5 \times 49.5 \times 7500^4}{384 \times 4700\sqrt{27.6} \times 4.3 \times 10^9} = 19.2mm$$

To find long term deflection first we find $\Delta_{i_{sus}}$

$\Delta_i = 19.2mm$ $\qquad$ $49.5kN$

$\Delta_{i_{sus}} = ?\ mm$ $\qquad$ $29kN$

$\Delta_{i_{sus}} = 11.25mm$

$\xi = 2$

$$\lambda = \frac{\xi}{1 + 50\rho'} = \frac{2}{1 + (50)0} = 2$$

$\Delta_{(cp+sh)}$ =long term deflection=$\Delta_{sus} = \lambda \Delta_{i_{sus}} = 2 \times 11.25 = 22.5mm$

Total deflection = Immediate Deflection + Long-Term Deflection

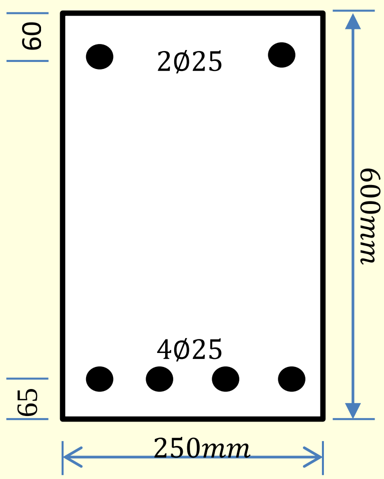
$$\Delta_{total} = \Delta_i + \Delta_{sus} = 19.2 + 22.5 = 41.7\ mm$$

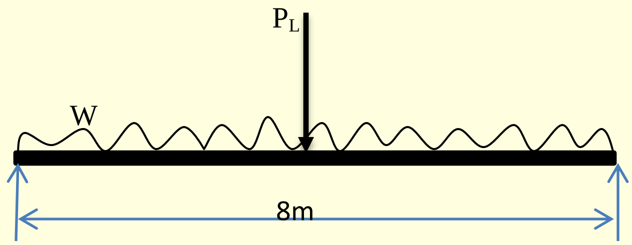
Example

A simply supported beam carries a dead load of 10kN/m including self-weight and concentrated load in the middle of the span as shown in the figure below.

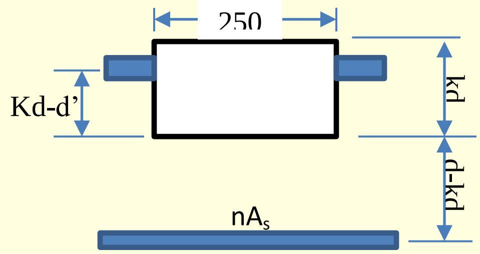
Consider $f'_c = 21MPa, f_y = 400MPa \ and \ n = 9$

1- Find the maximum concentrated load (P_L).
2- Find the instantaneous deflection at the middle of span due to dead and live loads.





$$M_a = \frac{Wl^2}{8} + \frac{PL}{4}$$



$$A_s = 4\frac{25^2\pi}{4} = 1936mm^2$$

$$A'_s = 2\frac{25^2\pi}{4} = 982mm^2$$

$$B = \frac{b}{nA_s} = 0.01434$$

$$r = (2n-1)\frac{A'_s}{nA_s} = 0.958$$

$$kd = \frac{\left[\sqrt{2dB\left(1 + r\frac{d'}{d}\right) + (1+r)^2} - (1+r)\right]}{B} = 182mm$$

$$I_{cr} = \frac{b(kd)^3}{3} + (2n-1)A'_s(kd-d')^2 + nA_s(d-kd)^2 = 2.952 \times 10^9 mm^4$$

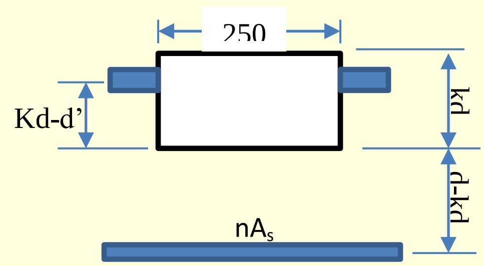
$$f_c = \frac{Mc}{I_{cr}} = 0.45(21) = \frac{M \times 10^6 \times 182}{2.952 \times 10^9} \rightarrow M = 153.27kN.m$$

$$f_s = n\frac{Mc}{I_{cr}} = 170 = 9 \times \frac{M \times 10^6 \times (535 - 182)}{2.952 \times 10^9} \rightarrow M = 158kN.m$$

$$f'_s = 2n\frac{Mc}{I_{cr}} = 170 = 2 \times 9\frac{M \times 10^6 \times (182 - 60)}{2.952 \times 10^9} \rightarrow M = 228.7kN.m$$

Chose M=153.27kN.m

$$M_a = \frac{Wl^2}{8} + \frac{PL}{4} \rightarrow 153.27 = \frac{10 \times 8^2}{8} + \frac{P_L \times 8}{4} \rightarrow P_L = 36.635kN$$



$$I_g = \frac{bh^3}{12} = \frac{250(600)^3}{12} = 4.5 \times 10^9 mm^4$$

$$f_{cr} = \frac{M_{cr} \times 10^6 \times c_t}{I_g}$$

$$0.62\sqrt{21} = \frac{M_{cr} \times 10^6 \times 300}{4.5 \times 10^9} \quad \rightarrow \quad M_{cr} = 42.6kN.m$$

$$B = \frac{b}{nA_s} = 0.01434$$

$$r = (n-1)\frac{A'_s}{nA_s} = 0.45$$

$$kd = \frac{\left[\sqrt{2dB\left(1 + r\frac{d'}{d}\right) + (1+r)^2} - (1+r)\right]}{B} = 198mm$$

$$I_{cr} = \frac{b(kd)^3}{3} + (n-1)A'_s(kd - d')^2 + nA_s(d - kd)^2 = 2.803 \times 10^9 mm^4$$

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr}$$

$$= \left(\frac{42.6}{153.27}\right)^3 4.5 \times 10^9 + \left[1 - \left(\frac{42.6}{153.27}\right)^3\right] 2.803 \times 10^9$$

$$= 2.84 \times 10^9 mm^4$$

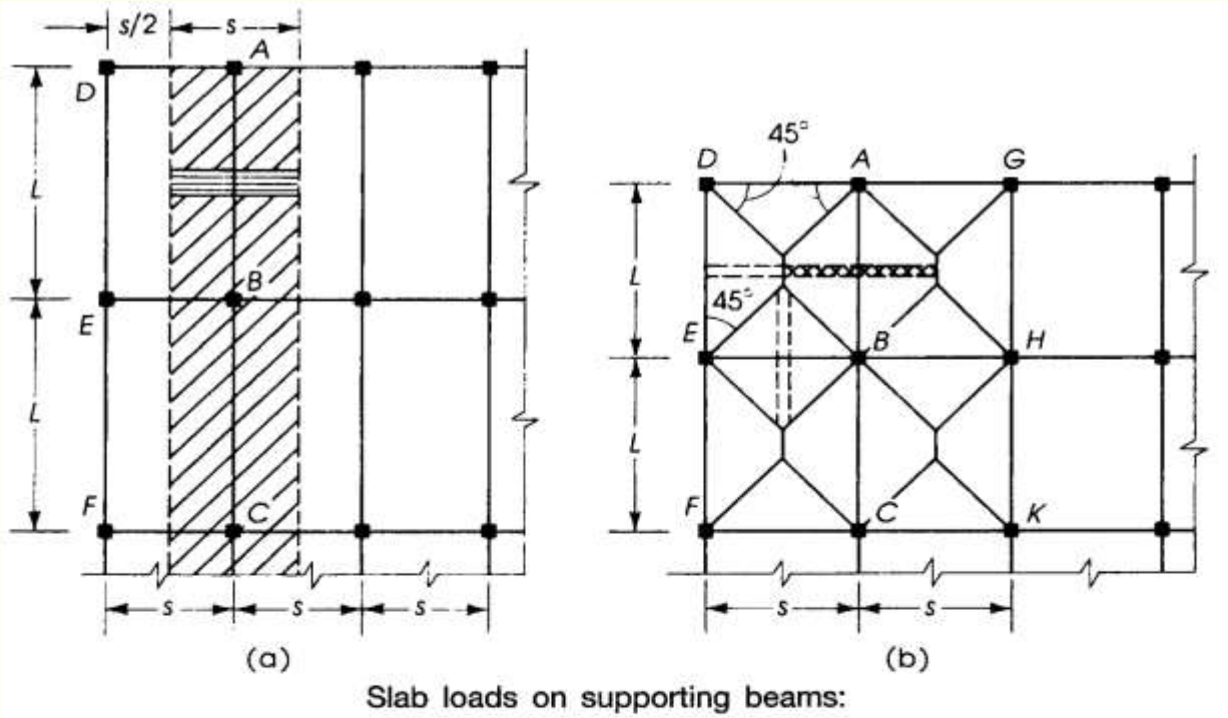
$$\Delta_i = \frac{5Wl^4}{384E_cI_e} + \frac{PL^3}{48E_cI_e} = \frac{5 \times 10 \times 8000^4}{348 \times 4700\sqrt{21} \times 2.803 \times 10^9} + \frac{36.635 \times 10^3 \times 8000^3}{48 \times 4700\sqrt{21} \times 2.803 \times 10^9} =$$

CHAPTER 9

TWO-WAY SLABS

9.1 INTRODUCTION

When the slab is supported on all four sides and the length, L , is less than twice the width, S , the slab will deflect in two directions, and the loads on the slab are transferred to all four supports. This slab is referred to as a two-way slab. The bending moments and deflections in such slabs are less than those in one-way slabs; thus, the same slab can carry more load when supported on four sides. The load in this case is carried in two directions, and the bending moment in each direction is much less than the bending moment in the slab if the load were carried in one direction only.



Slab loads on supporting beams:

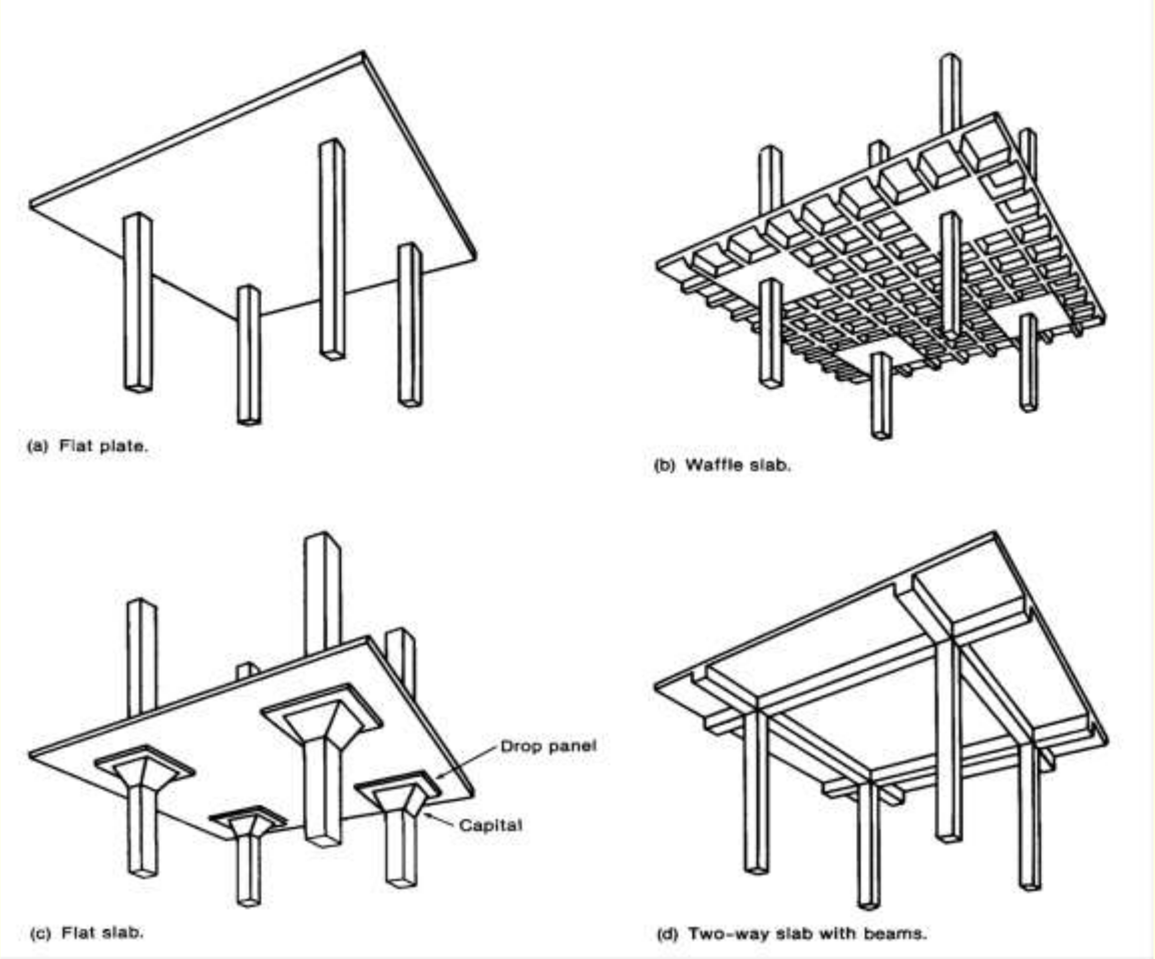
(a) one-way direction, $L/s > 2$; (b) two-way direction, $L/s \leq 2$.

9.2 TYPES OF TWO-WAY SLABS

Structural two-way concrete slabs may be classified as follows:

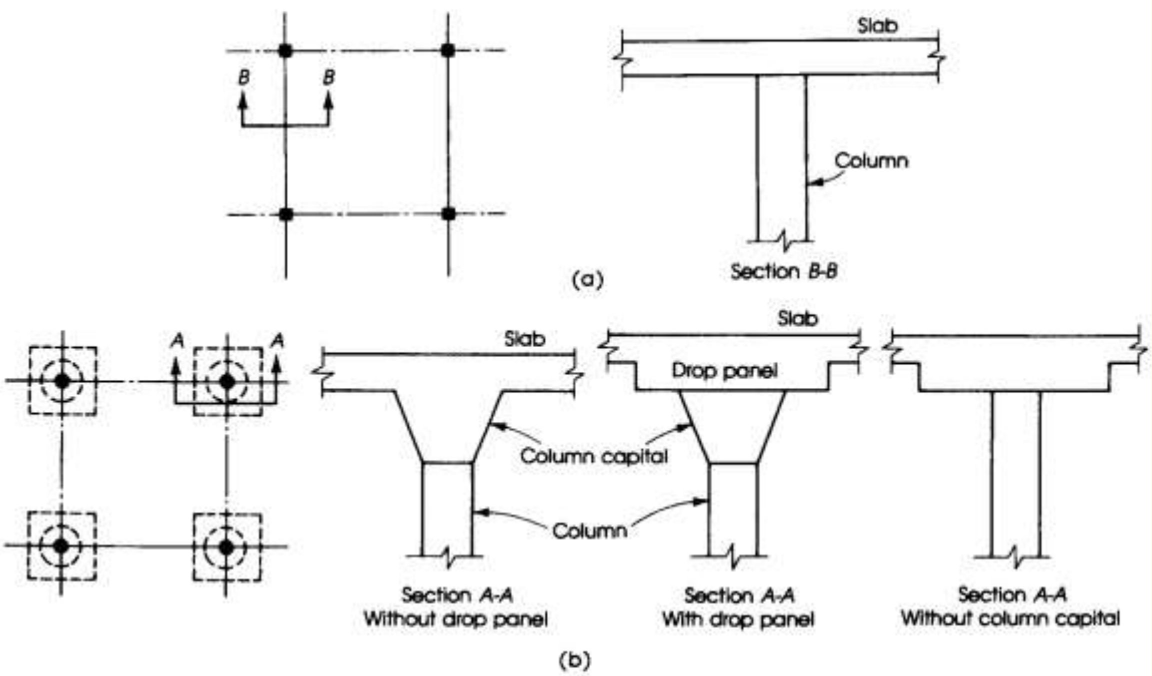
1. **Two-Way Slabs on Beams:** This case occurs when the two-way slab is supported by beams on all four sides. The loads from the slab are transferred to all four supporting beams, which, in turn, transfer the loads to the columns.
2. **Flat Slabs**: A flat slab is a two-way slab reinforced in two directions that usually does not have beams or girders, and the loads are transferred directly to the supporting columns. The column lends to punch through the slab, which can be treated by three methods

a. Using a drop panel and a column capital.
b. Using a drop panel without a column capital. The concrete panel around the column capital should be thick enough to withstand the diagonal tensile stresses arising from the punching shear.
c. Using a column capital without drop panel, which is not common.



(a) Flat plate. (b) Waffle slab. (c) Flat slab. (d) Two-way slab with beams.

3. **Flat-Plate Floors:** A flat-plate floor is a two-way slab system consisting of a uniform slab that rests directly on columns and does not have beams or column capitals (Fig. a). In this case the column tends to punch through the slab, producing diagonal tensile stresses. Therefore, a general increase in the slab thickness is required or special reinforcement is used.



Two-way slabs without beams:

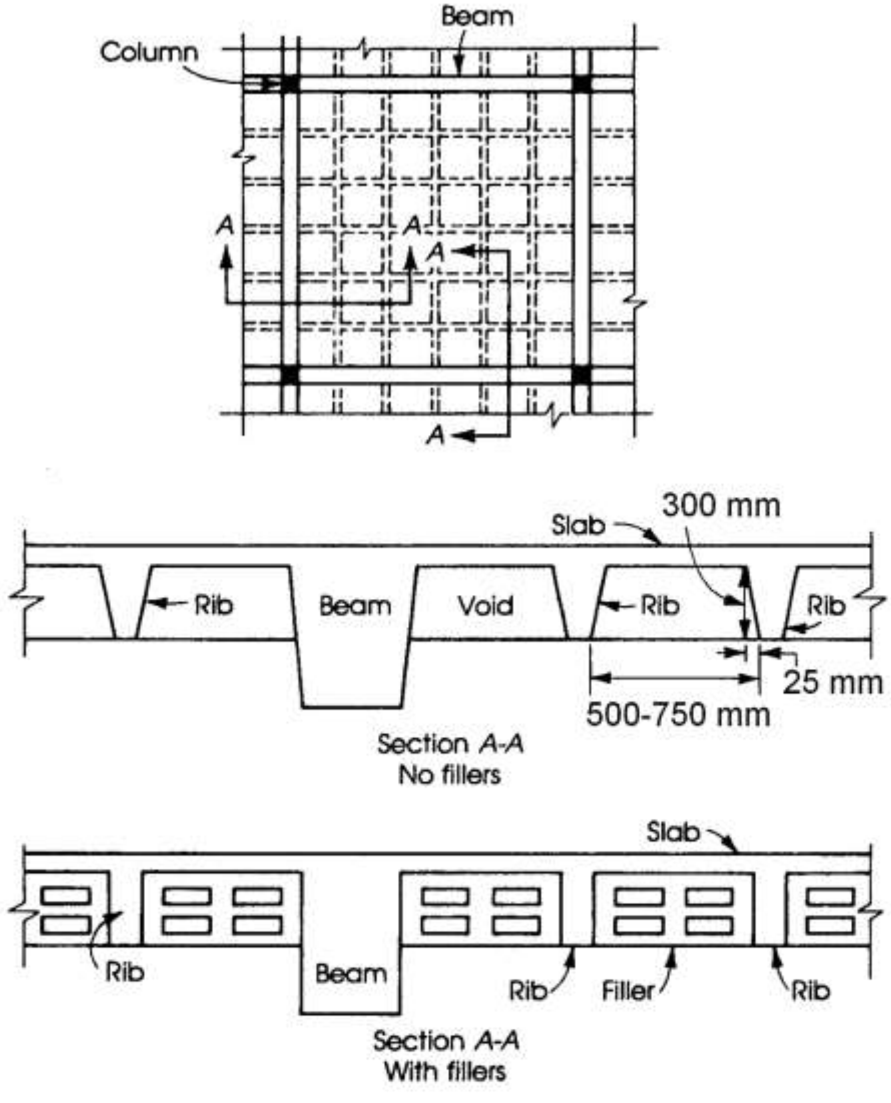
(*a*) flat plate floor and section; (*b*) flat slab floor and sections; (c) ribbed slab and sections.

4. **Two-Way Ribbed Slabs and the Waffle Slab System**: This type of slab consists of a floor slab with a length-to-width ratio less than 2. The thickness of the slab is usually 5 to 10 cm and is supported by ribs (or joists) in two directions. The ribs are arranged in each direction at spacing of about$50\ cm\ to\ 75\ cm$, producing square or rectangular shapes. The ribs can also be arranged at 45°or 60° from the centerline of slabs, producing architectural shapes at the soffit of the slab. In two-way ribbed slabs, different systems can be adopted:

a. A two-way rib system with voids between the ribs, obtained by using special removable and usable forms (pans) that are normally square in shape. The ribs are supported on four sides by girders that rest on columns. This type is called a two-way ribbed (joist) slab system.

b. A two-way rib system with permanent fillers between ribs that produce horizontal slab soffits. The fillers may be of hollow, lightweight or normal-weight concrete or any other lightweight material. The ribs are supported by girders on four sides, which in turn are supported by columns. This type is also called a two-way ribbed (joist) slab system or a hollow-block two-way ribbed system.

c. A two-way rib system with voids between the ribs with the ribs continuing in both directions without supporting beams and resting directly on columns through solid panels above the columns. This type is called a waffle slab system.



(c)

9.3 ECONOMICAL CHOICE OF CONCRETE FLOOR SYSTEMS

Various types of floor systems can be used for general buildings, such as residential, office, and institutional buildings. The choice of an adequate and economic floor system depends on the type of building, architectural layout, aesthetic features, and the span length between columns. In general, the superimposed live load on buildings varies between 4 and $7\ kN/m^2$. A general guide for the economical use of floor systems can be summarized as follows:

1. **Flat Plates:** Flat plates are most suitable for spans of 6 to 8 m and live loads between $3\ and\ 5\ kN/m^2$. The advantages of adopting flat plates include low-cost formwork, exposed flat ceilings, and fast construction. Flat plates have low shear capacity and relatively low stiffness, which may cause noticeable deflection. Flat plates are widely used in buildings either as reinforced or prestressed concrete slabs.

2. **Flat Slabs:** Flat slabs are most suitable for spans of 6 to 9m and for live loads of $4\ and\ 7\ kN/m^2$. They need more formwork than flat plates, especially for column capitals. In most cases, only drop panels without column capitals are used.

3. **Waffle Slabs:** Waffle slabs are suitable for spans of 9 to 15m and live loads of$4\ and\ 7\ kN/m^2$. They carry heavier loads than flat plates and have attractive exposed ceilings. Formwork, including the use of pans, is quite expensive.

4. **Slabs on Beams:** Slabs on beams are suitable for spans between 6 to 9m and live loads of $3\ and\ 6\ kN/m^2$. The beams increase the stiffness of the slabs, producing relatively low deflection. Additional formwork for the beams is needed.

5. **One-Way Slabs on Beams:** One-way slabs on beams are most suitable for spans of 3 to 6m and a live load of $3\ and\ 5\ kN/m^2$. They can be used for larger spans with relatively higher cost and higher slab deflection. Additional formwork for the beams is needed.

6. **One-Way Joist Floor System:** A one-way joist floor system is most suitable for spans of 6 to 9m and live loads of $4\ and\ 6\ kN/m^2$. Because of the deep ribs, the concrete and steel quantities are relatively low, but expensive formwork is expected. The exposed ceiling of the slabs may look attractive.

9.4 MINIMUM THICKNESS OF TWO-WAY SLABS.

The ACI Code, Section 8.3.1 specifies a minimum slab thickness in two-way slabs to control deflection. The magnitude of a slab's deflection depends on many variables, including the flexural stiffness of the slab, which in turn is a function of the slab thickness ***h*** . By increasing the slab thickness, the flexural stiffness of the slab is increased, and consequently the slab deflection is reduced. Because the calculation of deflections in two-way slabs is complicated and to avoid excessive deflections, the ACI Code limits the thickness of these slabs by adopting the following three empirical limitations, which are based on experimental research. If these limitations are not met, it will be necessary to compute deflections.

1-For $0.2 \leq \alpha_{fm} \leq 2.0$

$$h = \frac{l_n\left(0.8 + \frac{f_y}{1400}\right)}{36 + 5\beta\left(\alpha_{fm} - 0.2\right)} \ldots\ldots\ldots (1)$$

but not less than 125 mm.

2-For $\alpha_{fm} > 2.0$

$$h = \frac{l_n\left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta} \ldots\ldots\ldots\ldots\ldots (2)$$

but not less than 90 mm.

3- For $\alpha_{fm} < 0.2$

$h = minimum\ slab\ thickness\ with\ out\ interior\ beams\ table$ 8.3.1.1

Where:

$l_n =$ clear span in the long direction measured face to face of columns

(or face to face of beams for slabs with beams).

$\beta =$ the ratio of the long to the short clear spans.

α_{fm} = the average value of α_f for all beams on the sides of a panel.

α_f = the ratio of flexural stiffness of a beam section $E_{cb}I_b$ to the flexural stiffness of the slab $E_{cs}I_s$, bounded laterally by the centerlines of the panels on each side of the beam.

$$\alpha_f = \frac{E_{cb}I_b}{E_{cs}I_s}$$

Where:
E_{cb} , and E_{cs} are the modulus of elasticity of concrete in the beam and the slab, respectively.
I_b =The gross moment of inertia of the beam section about the centroidal axis (the beam section includes a slab length on each side of the beam equal to the projection of the beam above or below the slab, whichever is greater, but not more than four times the slab thickness).
I_s =The moment of inertia of the gross section of the slab.

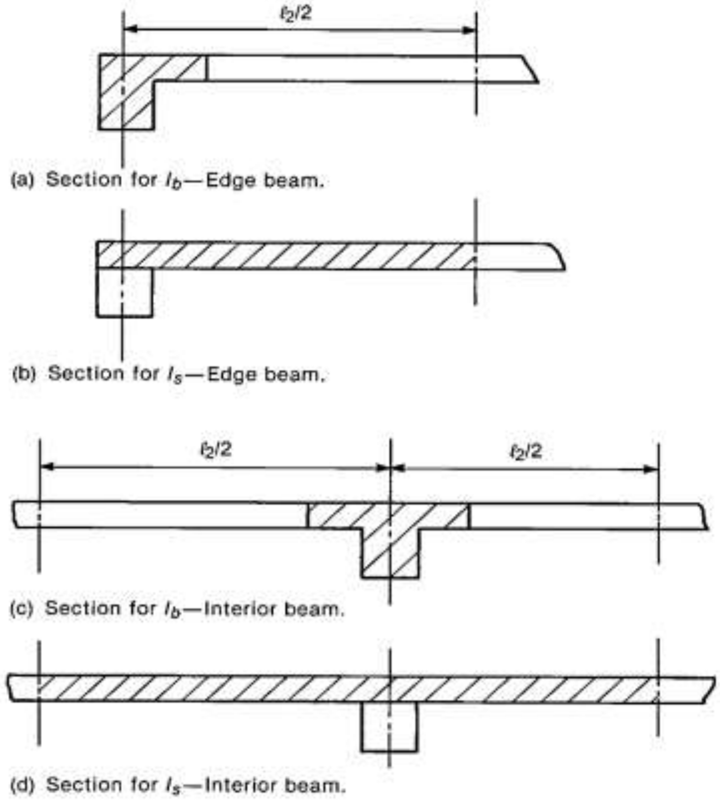
However, the thickness of any slab shall not be less than the following:
1. For slabs with $\alpha_{fm} \leq 2$ then thickness $\geq 125\ mm$.
2. For slabs with $\alpha_{fm} > 2$ then thickness $> 90\ mm$.

If no beams are used, as in the case of flat plates, then $\alpha_f = 0$ and $\alpha_{fm} = 0$. The ACI Code equations for calculating slab thickness, h take into account the effect of the span length, the panel shape, the steel reinforcement yield stress, f_y , and the flexural stiffness of beams. When very stiff beams are used, Eq. (1) may give a small slab thickness, and Eq. (2) may control. For flat plates and flat slabs, when no interior beams are used, the minimum slab thickness may be determined directly from Table 8.3.1.1 of the ACI Code, which is shown here.

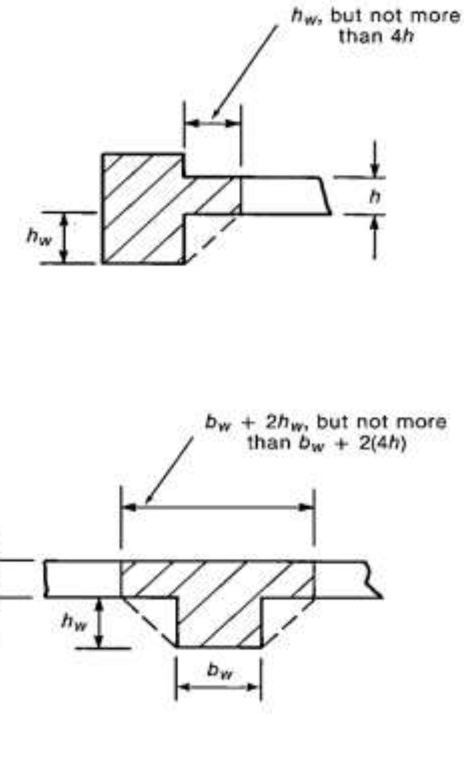
Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (mm)[1]

f_y, MPa[2]	Without drop panels[3]			With drop panels[3]		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams[4]		Without edge beams	With edge beams[4]	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
420	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

[1] ℓ_n is the clear span in the long direction, measured face-to-face of supports (mm).
[2] For f_y between the values given in the table, minimum thickness shall be calculated by linear interpolation.
[3] Drop panels as given in 8.2.4.
[4] Slabs with beams between columns along exterior edges. Exterior panels shall be considered to be without edge beams if α_f is less than 0.8. The value of α_f for the edge beam shall be calculated in accordance with 8.10.2.7.



Beam and slab sections for calculations of α_f



Cross section of beams as defined in ACI Code Section 13.2.4.

Other ACI Code limitations are summarized as follows:

1. For panels with discontinuous edges, end beams with a minimum α_f equal to 0.8 must be used; otherwise, the minimum slab thickness calculated by Eqs. (1) and (2) must be increased by at least 10% (ACI Code, Section. 8.3.1.2.1).

2. When drop panels are used without beams, the minimum slab thickness may be reduced by 10%. The drop panels should extend in each direction from the centerline of support a distance not less than one-sixth of the span length in that direction between center to center of supports and also project below the slab at least $h/4$. This reduction is included in Table 8.3.1.1.

3. Regardless of the values obtained by Eqs. (1) and (2), the thickness of two-way slabs shall not be less than the following:

(1) For slabs without beams or drop panels,125mm.
(2) For slabs without beams but with drop panels,100 mm.
(3) For slabs with beams on all four sides with, $\alpha_{fm} \geq 2.0$,90mm and for $\alpha_{fm} < 2.0$, 125mm (ACI Code, Section 8.3.1.1(a) and (b)).

The thickness of a slab also may be governed by shear. This is particularly likely if large moments are transferred to edge columns and for interior columns between two spans that are greatly different in length. The selection of slab thicknesses to satisfy shear requirements will be discussed later. Briefly, it is suggested that the trial slab thickness be chosen such that $V_u =\cong 0.5\ to\ 0.55(\emptyset V_v)$at edge columns and $V_u =\cong 0.85\ to\ 1.0(\emptyset V_v)$ at interior columns.

9.5 SLAB REINFORCEMENT REQUIREMENTS.

Placement Sequence.

In a flat plate or flat slab, the moments are larger in the slab strips spanning the long direction of the panels. As a result, the reinforcement for the long span generally is placed closer to the top and bottom of the slab than is the short-span reinforcement. This gives the larger effective depth for the larger moment. For slabs supported on beams having α_f greater than about 1.0, the opposite is true, and the reinforcing pattern should be reversed. If a particular placing sequence has been assumed in the reinforcement design, it should be shown or noted on the drawings. It also is important to maintain the same arrangements of layers throughout the entire floor, to avoid confusion in the field. Thus, if the east–west reinforcement is nearer the top and bottom surfaces in one area, this arrangement should be maintained over the entire slab, if at all possible.

Concrete Cover.

ACI Code Section 8.7.1.1 specifies the minimum clear cover to the surface of the reinforcement in slabs as $20mm\ for\ \emptyset 36$ and smaller bars, provided that the slab is not exposed to earth or to weather.

Spacing Requirements, Minimum Reinforcement, and Minimum Bar Size.

ACI Code Section 8.6.1.1requires that the minimum area of reinforcement provided for flexure should not be less than (**Table 8.6.1.1—*As,min***):

1. For slabs in which grade 280 ($f_y = 280MPa$) or 350 ($f_y = 350MPa$) deformed bars are used $\rho = 0.002$.
2. For slabs in which grade 420 ($f_y = 420MPa$) deformed bars or welded bars or welded wire fabric are used $\rho = 0.0018$.

The maximum spacing of reinforcement at critical design sections for positive and negative moments in both the middle and column strips shall not exceed two times the slab thickness (ACI Code Section 8.7.2.2), and the bar spacing shall not exceed at any location (ACI Code Section 24.4.3.3).

Although there is no code limit on bar size, the top steel bars add steps in slab should be enough to give adequate rigidity to prevent displacement of the bars under ordinary foot traffic before the concrete is placed.

Bar Cutoffs and Anchorages

For slabs without beams, ACI Code Section 8.7.4.1.3 allows the bars to be cut off as shown in the figure below (ACI Code Fig. 8.7.4.3.1a). Where adjacent spans have unequal lengths, the extension of the negative-moment bars past the face of the support is based on the length of the longer span.

ACI Code Section 8.7.4.1.1-a requires that the Positive moment reinforcement perpendicular to a discontinuous edge shall extend to the edge of slab and have embedment, straight or hooked,150mm at least in spandrel beams, columns, or walls.

ACI Code Section 8.7.4.1.1-b requires that all negative-moment steel perpendicular to an edge be bent, hooked, or otherwise anchored in spandrel beams, columns, and walls along the edge to develop $f_y$in tension. If there is no edge beam, this steel still should be hooked to act as torsional reinforcement and should extend to the minimum cover thickness from the edge of the slab.

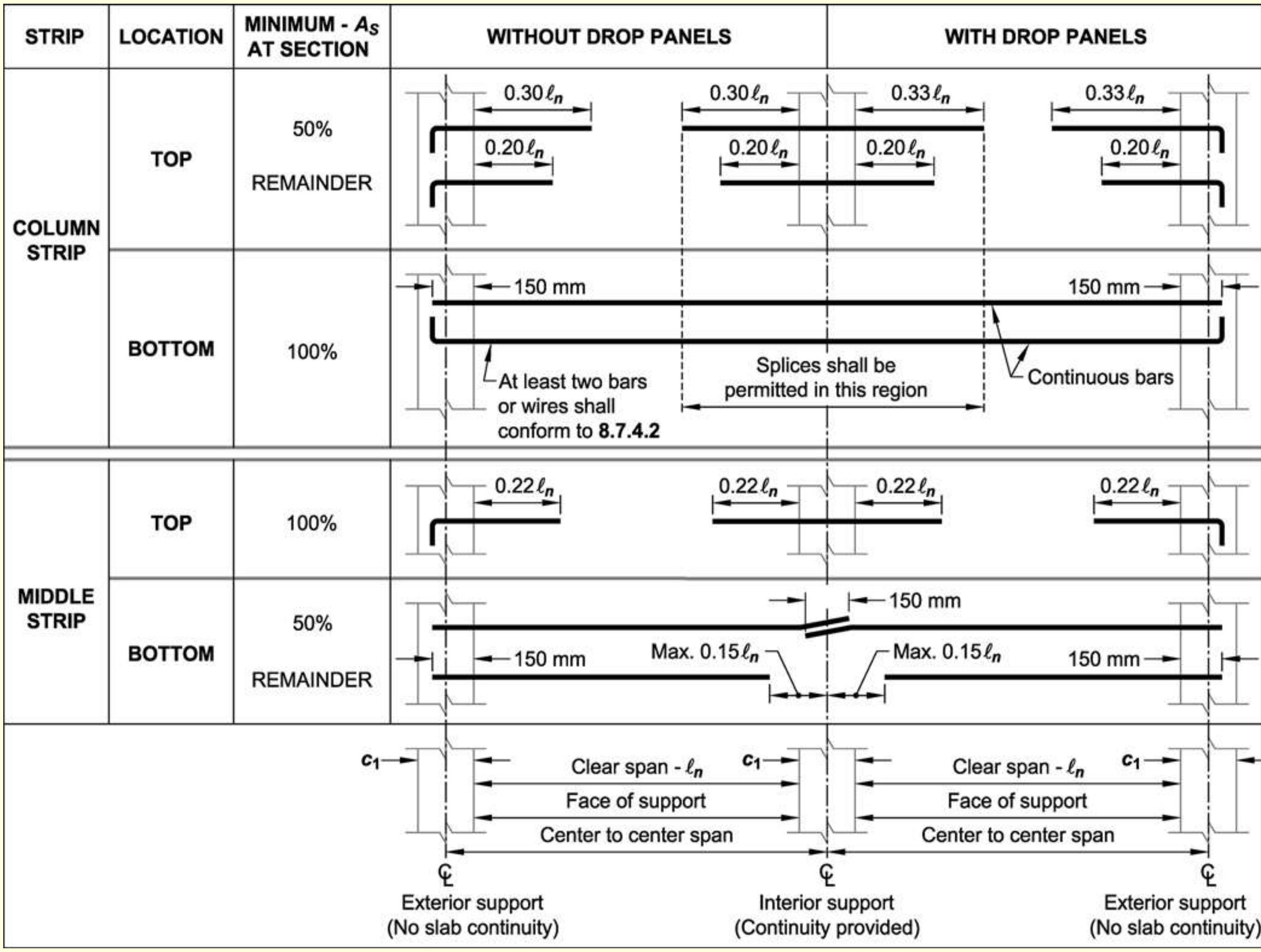


Fig. 8.7.4.1.3a*—Minimum extensions for deformed reinforcement in two-way slabs without beams.*

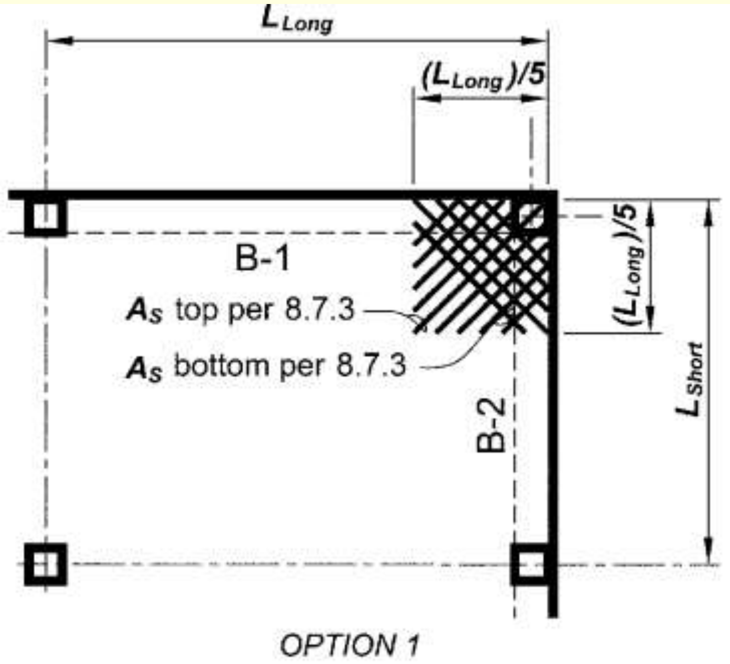
ACI Code Section 8.7.3.1 requires that at exterior corners of slabs supported by edge walls or where one or more edge beams have a value of greater than , top and bottom slab reinforcement shall be provided at exterior corners in accordance with 8.7.3.1.1 through 8.7.3.1.3.

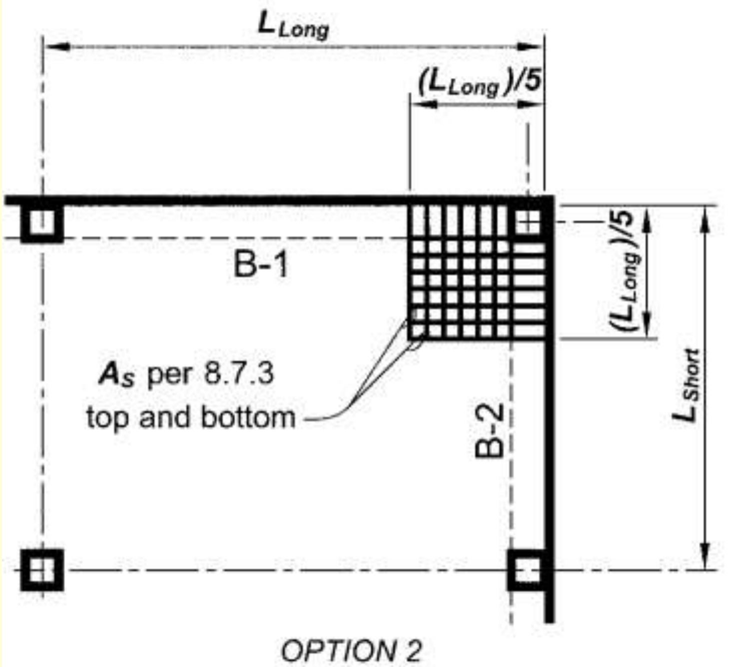
8.7.3.1.1 Factored moment due to corner effects, ***Mu***, shall be assumed to be about an axis perpendicular to the diagonal from the corner in the top of the slab and about an axis parallel to the diagonal from the corner in the bottom of the slab.

8.7.3.1.2 Reinforcement shall be provided for a distance in each direction from the corner equal to one fifth the longer span.

8.7.3.1.3 Reinforcement shall be placed parallel to the diagonal in the top of the slab and perpendicular to the diagonal in the bottom of the slab. Alternatively, reinforcement shall be placed in two layers parallel to the sides of the slab in both the top and bottom of the slab.

R8.7.3.1 Unrestrained corners of two-way slabs tend to lift when loaded. If this lifting tendency is restrained by edge walls or beams, bending moments result in the slab. This section requires reinforcement to resist these moments and control cracking. Reinforcement provided for flexure in the primary directions may be used to satisfy this requirement. Refer to Fig. R8.7.3.1.





Notes:

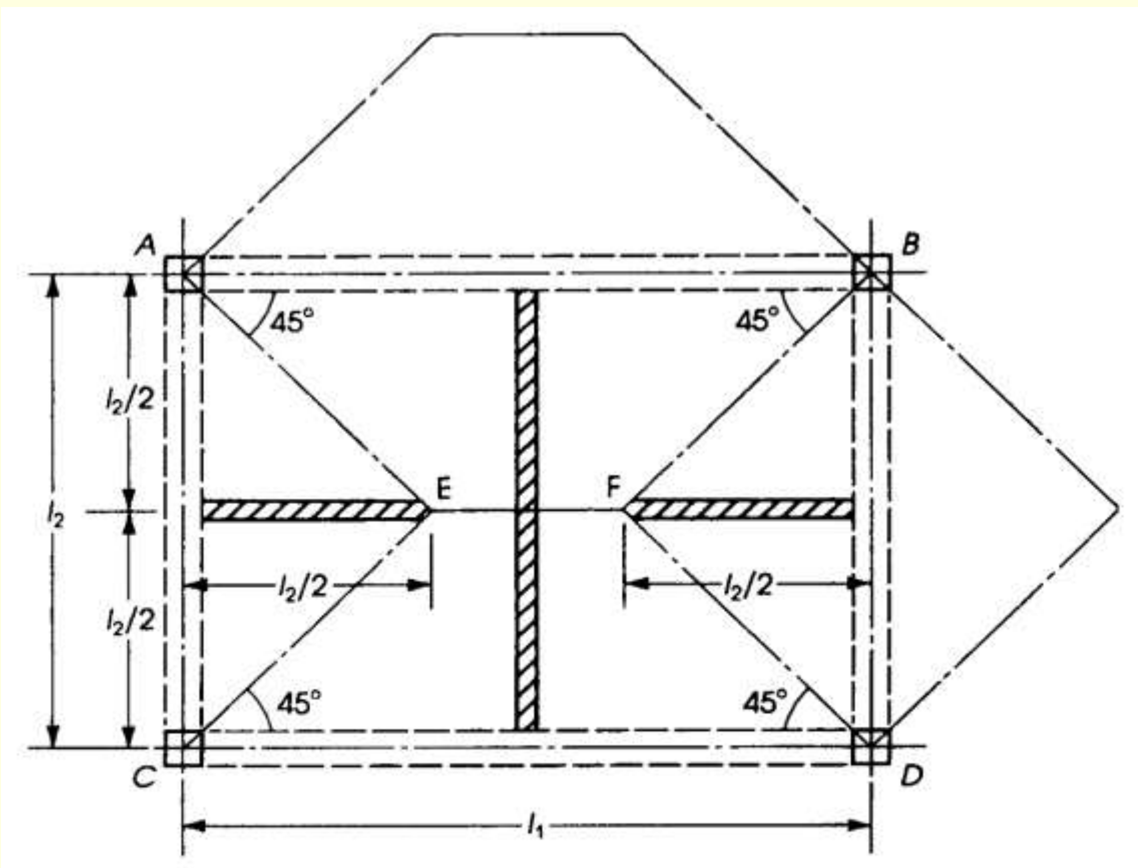
1. Applies where B-1 or B-2 has $\alpha_f > 1.0$
2. Max. bar spacing $2h$, where h = slab thickness.

9.6 SHEAR STRENGTH OF TWO-WAY SLABS.

In a two-way floor system, the slab must have adequate thickness to resist both bending moments and shear forces at the critical sections. To investigate the shear capacity of two-way slabs, the following cases should be considered.

9.6.1 Two-Way Slabs Supported on Beams

In two-way slabs supported on beams, the critical sections are at a distance from the face of the supporting beams, and the shear capacity of each section is $\emptyset V_c = \emptyset \frac{1}{6}\sqrt{f'_c} b_w d$. When the supporting beams are stiff and are capable of transmitting floor loads to the columns, they are assumed to carry loads acting on floor areas bounded by 45° lines drawn from the corners, as shown in the figure below. The loads on the trapezoidal areas will be carried by the long beams AB and CD, whereas the loads on the triangular areas will be carried by the short beams AC and BD . The shear per unit width of slab is highest between E and F in both directions, and $V_u = W_u l_2/2$, where is the uniform factored load per unit area.



Areas supported by beams in two-way slab floor system.

If no shear reinforcement is provided, the shearing force at a distance d from the face of the beam, V_{ud} must be equal to

$$V_{ud} \leq \emptyset V_c = \emptyset \frac{1}{6} \lambda \sqrt{f'_c} b_w d$$

Where:

$$V_{ud} = W_u = \left(\frac{l_2}{2} - \frac{b_w}{2} - d\right)$$

9.7 ANALYSIS AND DESIGN OF TWO-WAY SLABS.

An exact analysis of forces and displacements in a two-way slab is complex, due to its highly indeterminate nature; this is true even when the effects of creep and nonlinear behavior of the concrete are neglected. Numerical methods such as finite elements can be used, but simplified methods such as those presented by the ACI Code are more suitable for practical design. **The ACI Code, Chapter 8**, assumes that the slabs behave as wide, shallow beams that form, with the columns above and below them, a rigid frame. The validity of this assumption of dividing the structure into equivalent frames has been verified by analytical and experimental research. It is also established that factored load capacity of two-way slabs with restrained boundaries is about twice that calculated by theoretical analysis, because a great deal of moment redistribution occurs in the slab before failure. At high loads, large deformations and deflections are expected; thus, a minimum slab thickness is required to maintain adequate deflection and cracking conditions under service loads.

The ACI Code specifies two methods for the design of two-way slabs:

1. The direct design method. DDM (ACI Code, Section 8.10), is an approximate procedure for the analysis and design of two-way slabs. It is limited to slab systems subjected to uniformly distributed loads and supported on equally or nearly equally spaced columns. The method uses a set of coefficients to determine the design moments at critical sections. Two-way slab systems that do not meet the limitations of the ACI Code, Section 8.10.2, must be analyzed by more accurate procedures.

2. The equivalent frame method, EFM (ACI Code, Section 8.11), is one in which a three-dimensional building is divided into a series of two-dimensional equivalent frames by cutting the building along lines midway between columns. The resulting frames are considered separately in the longitudinal and transverse directions of the building and treated floor by floor, as will be shown later.

The systems that do not meet the requirements permitting analysis by the "direct design method" of the present code, has led many engineers to continue to use the design method of the 1963 ACI Code (**The coefficient method**) for the special case of **two-way slabs supported on four sides of each slab panel by relatively deep, stiff, edge beams**. It has been used extensively here since 1963 for slabs supported at the edges by walls, steel beams, or monolithic concrete beams having a total depth **not less than about 3 times the slab thickness**. While it was not a part of the 1977 or later ACI Codes, its continued use is permissible under the current code provision (ACI Code 8.2) that a slab system may be designed by any procedure satisfying conditions of equilibrium and geometric compatibility, if it is shown that the design strength at every section is at least equal to the required strength, and that serviceability requirements are met.

9.8 SLAB ANALYSIS BY THE COEFFICIENT METHOD.

The coefficient method makes use of tables of moment coefficients for a variety of conditions. These coefficients are based on elastic analysis but also account for inelastic redistribution. In consequence, the design moment in either direction is smaller by an appropriate amount than the elastic maximum moment in that direction. The moments in the middle strips in the two directions are computed from

$$M_a = C_a w l_a^2$$

$$M_b = C_b w l_b^2$$

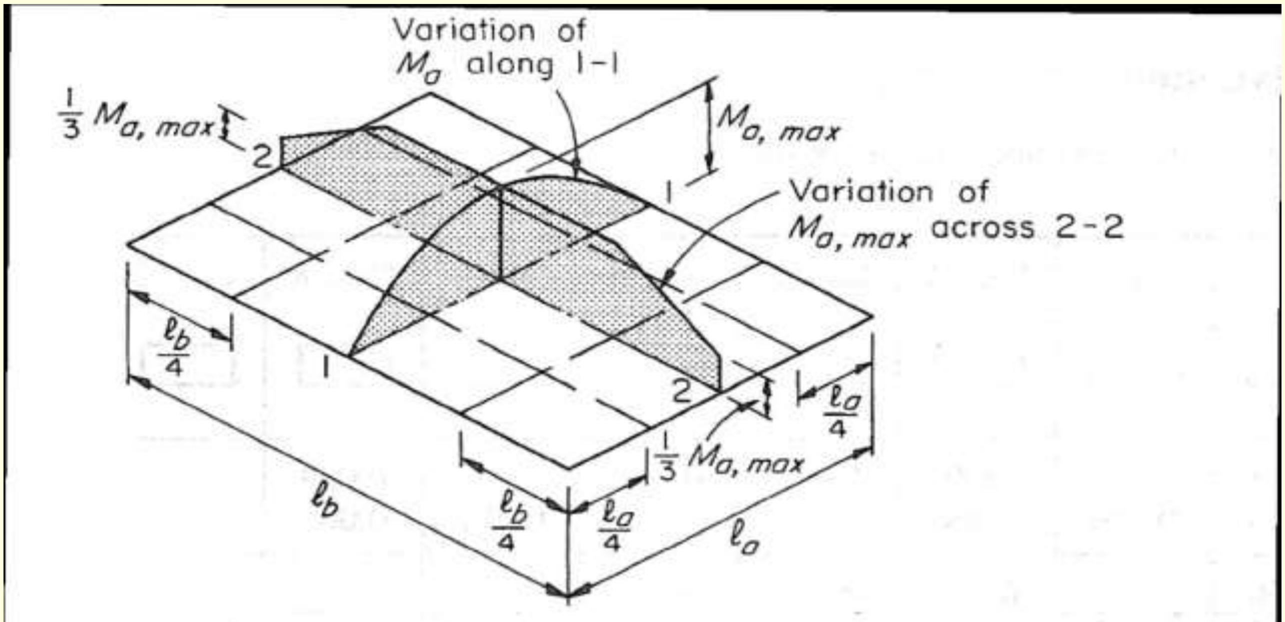
Where:

$C_a, C_b = tabulated\ moment\ coefficients.$

$w = uniform\ load\ kN/m^2$

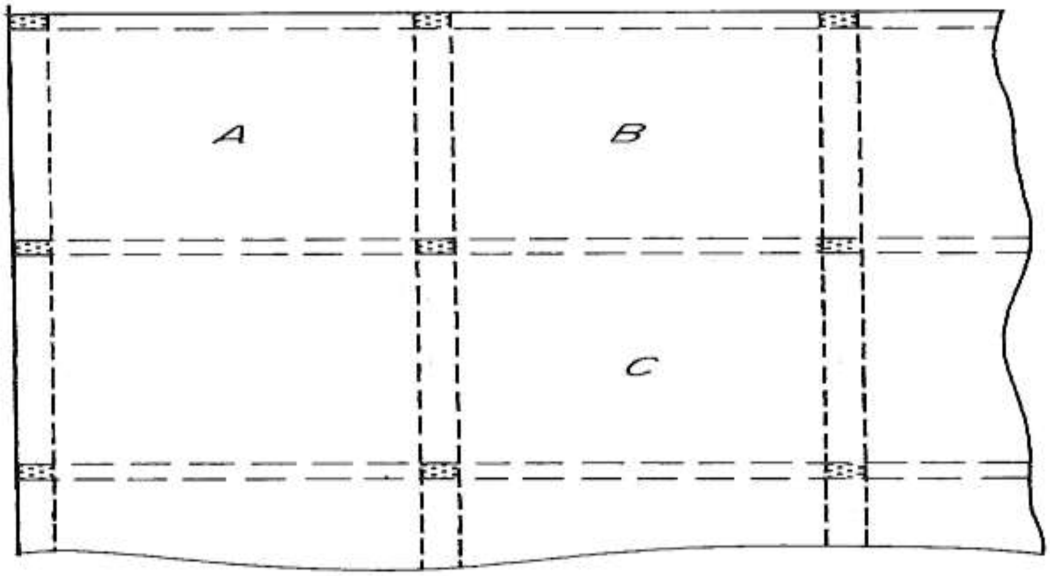
$l_a, l_b = length\ of\ clear\ span\ in\ the\ short\ and\ long\ directions\ respectively.$

The method provides that each panel be divided in both directions into a middle strip whose width is one-half that of the panel and two edge or column strips of one-quarter of the panel width (see figure below). The moments in both directions are larger in the center portion of the slab than in regions close to the edges. Correspondingly, it is provided that the entire middle strip be designed for the full, tabulated design moment. In the edge strips this moment is assumed to decrease from its full value at the edge of the middle strip to one-third of this value at the edge of the panel. This distribution is shown for the moments M_a in the short span direction in figure below. The lateral variation of the long span moments M_b is similar.



Variation of moments across the width of critical sections assumed for design.

The discussion so far has been restricted to a single panel simply supported at all four edges. An actual situation is shown in next figure, in which a system of beams supports a two-way slab. It is seen that some panels, such as A , have two discontinuous exterior edges, while the other edges are continuous with their neighbors. Panel B has one edge discontinuous and three continuous edges, the interior panel C has all edges continuous, and so on. At a continuous edge in a slab, moments are negative, just as at interior supports of continuous beams. Also, the magnitude of the positive moments depends on the conditions of continuity at all four edges.



Plan of a typical two-way slab floor with beams on column lines.

Correspondingly, Table 1 gives moment coefficients C , for negative moments at continuous edges. The details of the tables are self-explanatory. Maximum negative edge moments are obtained when both panels adjacent to the particular edge carry full dead and live load. Hence the moment is computed for this total load. Negative moments at discontinuous edges are assumed equal to one-third of the positive moments for the same direction. One must provide for such moments because some degree of restraint is generally provided at discontinuous edges by the torsional rigidity of the edge beam or by the supporting wall.

For positive moments there will be little, if any, rotation at the continuous edges if dead load alone is acting, because the loads on both adjacent panels tend to produce opposite rotations which cancel, or nearly so. For this condition, the continuous edges can be regarded as fixed, and the appropriate coefficients for the dead load positive moments are given in Table 2. On the other hand, the maximum live load positive moments are obtained when live load is placed only on the particular panel and not on any of the adjacent panels. In this case, some rotation will occur at all continuous edges. As an approximation it is assumed that there is 50% restraint for calculating these live load moments. The corresponding coefficients are given in Table 3. Finally, for computing shear in the slab and loads on the supporting beams, Table 4 gives the fractions of the total load that are transmitted in the two directions.

TABLE 1

Coefficients for negative moments in slabs[a]

$M_{a,neg} = C_{a,neg} w l_a^2$

$M_{b,neg} = C_{b,neg} w l_b^2$

where w = total uniform dead plus live load

Ratio $m = \frac{l_a}{l_b}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	$C_{a,neg}$		0.045		0.050	0.075	0.071		0.033	0.061
	$C_{b,neg}$		0.045	0.076	0.050			0.071	0.061	0.033
0.95	$C_{a,neg}$		0.050		0.055	0.079	0.075		0.038	0.065
	$C_{b,neg}$		0.041	0.072	0.045			0.067	0.056	0.029
0.90	$C_{a,neg}$		0.055		0.060	0.080	0.079		0.043	0.068
	$C_{b,neg}$		0.037	0.070	0.040			0.062	0.052	0.025
0.85	$C_{a,neg}$		0.060		0.066	0.082	0.083		0.049	0.072
	$C_{b,neg}$		0.031	0.065	0.034			0.057	0.046	0.021
0.80	$C_{a,neg}$		0.065		0.071	0.083	0.086		0.055	0.075
	$C_{b,neg}$		0.027	0.061	0.029			0.051	0.041	0.017
0.75	$C_{a,neg}$		0.069		0.076	0.085	0.088		0.061	0.078
	$C_{b,neg}$		0.022	0.056	0.024			0.044	0.036	0.014
0.70	$C_{a,neg}$		0.074		0.081	0.086	0.091		0.068	0.081
	$C_{b,neg}$		0.017	0.050	0.019			0.038	0.029	0.011
0.65	$C_{a,neg}$		0.077		0.085	0.087	0.093		0.074	0.083
	$C_{b,neg}$		0.014	0.043	0.015			0.031	0.024	0.008
0.60	$C_{a,neg}$		0.081		0.089	0.088	0.095		0.080	0.085
	$C_{b,neg}$		0.010	0.035	0.011			0.024	0.018	0.006
0.55	$C_{a,neg}$		0.084		0.092	0.089	0.096		0.085	0.086
	$C_{b,neg}$		0.007	0.028	0.008			0.019	0.014	0.005
0.50	$C_{a,neg}$		0.086		0.094	0.090	0.097		0.089	0.088
	$C_{b,neg}$		0.006	0.022	0.006			0.014	0.010	0.003

[a] A crosshatched edge indicates that the slab continues across, or is fixed at, the support; an unmarked edge indicates a support at which torsional resistance is negligible.

TABLE 2

Coefficients for dead load positive moments in slabs[a]

$M_{a,pos,dl} = C_{a,dl} w l_a^2$ where $w =$ total uniform dead load

$M_{b,pos,dl} = C_{b,dl} w l_b^2$

Ratio $m = \frac{l_a}{l_b}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	$C_{a,dl}$	0.036	0.018	0.018	0.027	0.027	0.033	0.027	0.020	0.023
	$C_{b,dl}$	0.036	0.018	0.027	0.027	0.018	0.027	0.033	0.023	0.020
0.95	$C_{a,dl}$	0.040	0.020	0.021	0.030	0.028	0.036	0.031	0.022	0.024
	$C_{b,dl}$	0.033	0.016	0.025	0.024	0.015	0.024	0.031	0.021	0.017
0.90	$C_{a,dl}$	0.045	0.022	0.025	0.033	0.029	0.039	0.035	0.025	0.026
	$C_{b,dl}$	0.029	0.014	0.024	0.022	0.013	0.021	0.028	0.019	0.015
0.85	$C_{a,dl}$	0.050	0.024	0.029	0.036	0.031	0.042	0.040	0.029	0.028
	$C_{b,dl}$	0.026	0.012	0.022	0.019	0.011	0.017	0.025	0.017	0.013
0.80	$C_{a,dl}$	0.056	0.026	0.034	0.039	0.032	0.045	0.045	0.032	0.029
	$C_{b,dl}$	0.023	0.011	0.020	0.016	0.009	0.015	0.022	0.015	0.010
0.75	$C_{a,dl}$	0.061	0.028	0.040	0.043	0.033	0.048	0.051	0.036	0.031
	$C_{b,dl}$	0.019	0.009	0.018	0.013	0.007	0.012	0.020	0.013	0.007
0.70	$C_{a,dl}$	0.068	0.030	0.046	0.046	0.035	0.051	0.058	0.040	0.033
	$C_{b,dl}$	0.016	0.007	0.016	0.011	0.005	0.009	0.017	0.011	0.006
0.65	$C_{a,dl}$	0.074	0.032	0.054	0.050	0.036	0.054	0.065	0.044	0.034
	$C_{b,dl}$	0.013	0.006	0.014	0.009	0.004	0.007	0.014	0.009	0.005
0.60	$C_{a,dl}$	0.081	0.034	0.062	0.053	0.037	0.056	0.073	0.048	0.036
	$C_{b,dl}$	0.010	0.004	0.011	0.007	0.003	0.006	0.012	0.007	0.004
0.55	$C_{a,dl}$	0.088	0.035	0.071	0.056	0.038	0.058	0.081	0.052	0.037
	$C_{b,dl}$	0.008	0.003	0.009	0.005	0.002	0.004	0.009	0.005	0.003
0.50	$C_{a,dl}$	0.095	0.037	0.080	0.059	0.039	0.061	0.089	0.056	0.038
	$C_{b,dl}$	0.006	0.002	0.007	0.004	0.001	0.003	0.007	0.004	0.002

[a] A crosshatched edge indicates that the slab continues across, or is fixed at, the support; an unmarked edge indicates a support at which torsional resistance is negligible.

TABLE 3

Coefficients for live load positive moments in slabs[a]

$M_{a,pos,ll} = C_{a,ll} w l_a^2$
$M_{b,pos,ll} = C_{b,ll} w l_b^2$ where w = total uniform live load

Ratio $m = \frac{l_a}{l_b}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	$C_{a,ll}$	0.036	0.027	0.027	0.032	0.032	0.035	0.032	0.028	0.030
	$C_{b,ll}$	0.036	0.027	0.032	0.032	0.027	0.032	0.035	0.030	0.028
0.95	$C_{a,ll}$	0.040	0.030	0.031	0.035	0.034	0.038	0.036	0.031	0.032
	$C_{b,ll}$	0.033	0.025	0.029	0.029	0.024	0.029	0.032	0.027	0.025
0.90	$C_{a,ll}$	0.045	0.034	0.035	0.039	0.037	0.042	0.040	0.035	0.036
	$C_{b,ll}$	0.029	0.022	0.027	0.026	0.021	0.025	0.029	0.024	0.022
0.85	$C_{a,ll}$	0.050	0.037	0.040	0.043	0.041	0.046	0.045	0.040	0.039
	$C_{b,ll}$	0.026	0.019	0.024	0.023	0.019	0.022	0.026	0.022	0.020
0.80	$C_{a,ll}$	0.056	0.041	0.045	0.048	0.044	0.051	0.051	0.044	0.042
	$C_{b,ll}$	0.023	0.017	0.022	0.020	0.016	0.019	0.023	0.019	0.017
0.75	$C_{a,ll}$	0.061	0.045	0.051	0.052	0.047	0.055	0.056	0.049	0.046
	$C_{b,ll}$	0.019	0.014	0.019	0.016	0.013	0.016	0.020	0.016	0.013
0.70	$C_{a,ll}$	0.068	0.049	0.057	0.057	0.051	0.060	0.063	0.054	0.050
	$C_{b,ll}$	0.016	0.012	0.016	0.014	0.011	0.013	0.017	0.014	0.011
0.65	$C_{a,ll}$	0.074	0.053	0.064	0.062	0.055	0.064	0.070	0.059	0.054
	$C_{b,ll}$	0.013	0.010	0.014	0.011	0.009	0.010	0.014	0.011	0.009
0.60	$C_{a,ll}$	0.081	0.058	0.071	0.067	0.059	0.068	0.077	0.065	0.059
	$C_{b,ll}$	0.010	0.007	0.011	0.009	0.007	0.008	0.011	0.009	0.007
0.55	$C_{a,ll}$	0.088	0.062	0.080	0.072	0.063	0.073	0.085	0.070	0.063
	$C_{b,ll}$	0.008	0.006	0.009	0.007	0.005	0.006	0.009	0.007	0.006
0.50	$C_{a,ll}$	0.095	0.066	0.088	0.077	0.067	0.078	0.092	0.076	0.067
	$C_{b,ll}$	0.006	0.004	0.007	0.005	0.004	0.005	0.007	0.005	0.004

[a] A crosshatched edge indicates that the slab continues across, or is fixed at, the support; an unmarked edge indicates a support at which torsional resistance is negligible.

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TABLE 4

Ratio of load W in l_a and l_b directions for shear in slab and load on supports[a]

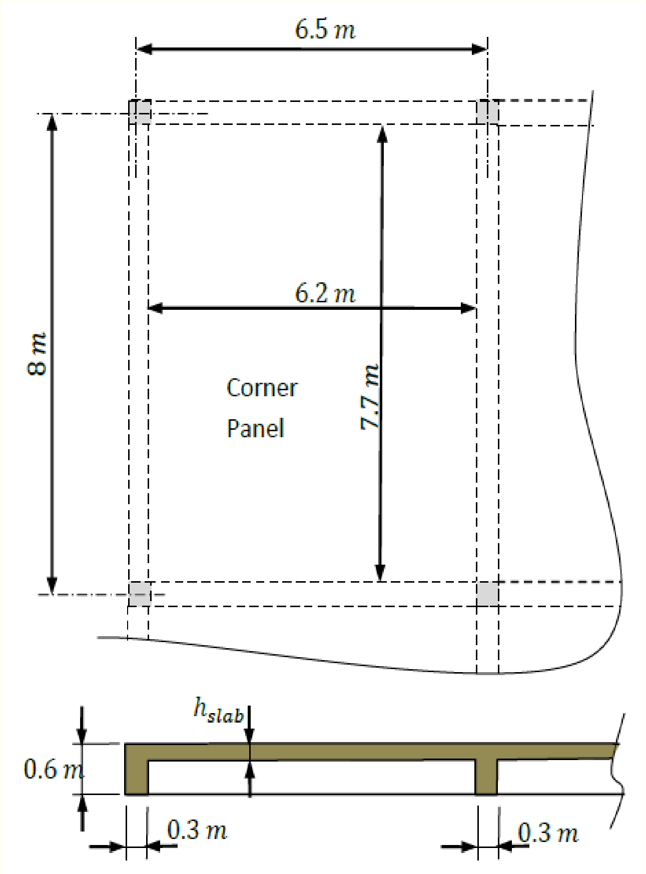
Ratio $m = \frac{l_a}{l_b}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	W_a	0.50	0.50	0.17	0.50	0.83	0.71	0.29	0.33	0.67
	W_b	0.50	0.50	0.83	0.50	0.17	0.29	0.71	0.67	0.33
0.95	W_a	0.55	0.55	0.20	0.55	0.86	0.75	0.33	0.38	0.71
	W_b	0.45	0.45	0.80	0.45	0.14	0.25	0.67	0.62	0.29
0.90	W_a	0.60	0.60	0.23	0.60	0.88	0.79	0.38	0.43	0.75
	W_b	0.40	0.40	0.77	0.40	0.12	0.21	0.62	0.57	0.25
0.85	W_a	0.66	0.66	0.28	0.66	0.90	0.83	0.43	0.49	0.79
	W_b	0.34	0.34	0.72	0.34	0.10	0.17	0.57	0.51	0.21
0.80	W_a	0.71	0.71	0.33	0.71	0.92	0.86	0.49	0.55	0.83
	W_b	0.29	0.29	0.67	0.29	0.08	0.14	0.51	0.45	0.17
0.75	W_a	0.76	0.76	0.39	0.76	0.94	0.88	0.56	0.61	0.86
	W_b	0.24	0.24	0.61	0.24	0.06	0.12	0.44	0.39	0.14
0.70	W_a	0.81	0.81	0.45	0.81	0.95	0.91	0.62	0.68	0.89
	W_b	0.19	0.19	0.55	0.19	0.05	0.09	0.38	0.32	0.11
0.65	W_a	0.85	0.85	0.53	0.85	0.96	0.93	0.69	0.74	0.92
	W_b	0.15	0.15	0.47	0.15	0.04	0.07	0.31	0.26	0.08
0.60	W_a	0.89	0.89	0.61	0.89	0.97	0.95	0.76	0.80	0.94
	W_b	0.11	0.11	0.39	0.11	0.03	0.05	0.24	0.20	0.06
0.55	W_a	0.92	0.92	0.69	0.92	0.98	0.96	0.81	0.85	0.95
	W_b	0.08	0.08	0.31	0.08	0.02	0.04	0.19	0.15	0.05
0.50	W_a	0.94	0.94	0.76	0.94	0.99	0.97	0.86	0.89	0.97
	W_b	0.06	0.06	0.24	0.06	0.01	0.03	0.14	0.11	0.03

[a] A crosshatched edge indicates that the slab continues across, or is fixed at, the support; an unmarked edge indicates a support at which torsional resistance is negligible.

Example (Design of two-way edge-supported solid slab):

A monolithic reinforced concrete floor is to be composed of rectangular bays measuring $6.5 \times 8\ m$, as shown. Beams of width $30\ cm$ and depth $60\ cm$ are provided on all column lines; thus the clear-span dimensions for the two-way slab panels are $6.2 \times 7.7\ m$. The floor is to be designed to carry a service live load $5\ kN/m^2$ and a dead load on the slab due to self-weight plus weight of :

- Tiles= $3\ cm$.
- Mortar =2 cm .
- Sand =7 cm.
- Plaster=2 cm.
- Partitions=5 kN/m^2.



Given:$f'_c = 20\ MPa\ and\ f_y = 400\ MPa.$

Find the required slab thickness and reinforcement for the corner panel shown.

Solution:

1. Minimum thickness (deflection requirements):

For slabs of this type the first trial thickness is often taken equal to

$$h = \frac{panel\ parameter}{180} = \frac{2(7.7 + 6.2)}{180}$$

=0.154m take h=16 cm.

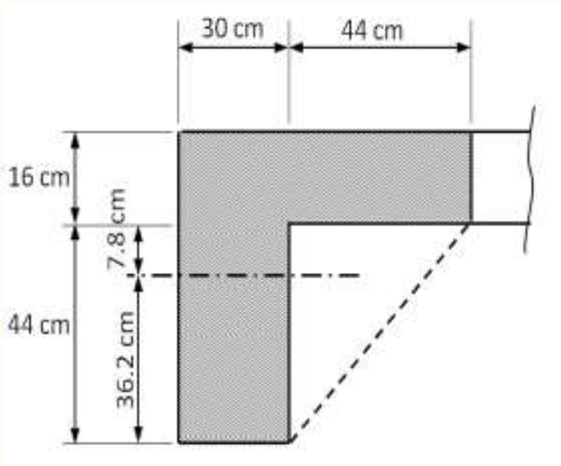
Check for the minimum thickness of the slab:

- Exterior beam:

$$h_w = 44\ cm\ < 4h = 4 \times 16 = 64\ cm\ OK$$

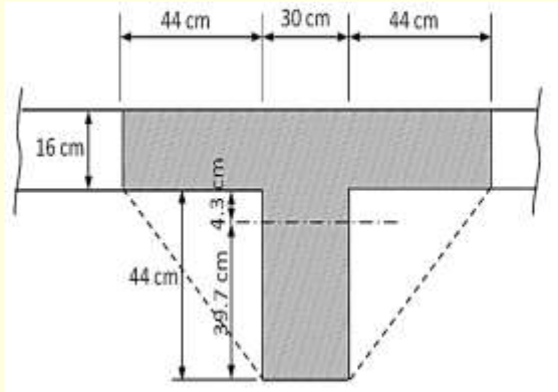
$$y_c = \frac{16(30+44)\left(44+\frac{16}{2}\right) + 30 \times 44 \times \frac{44}{2}}{16(30+44) + 30 \times 44} =$$

=36.2 cm



$$I_b = \frac{(30+44)(16+7.8)^3}{3} - \frac{44 \times 7.8^3}{3} + 30\frac{36.2^3}{3} = 799957\ cm^4$$

- Interior beam:



$b_w + 2h_w = 30 + 2(44) = 118\ cm$

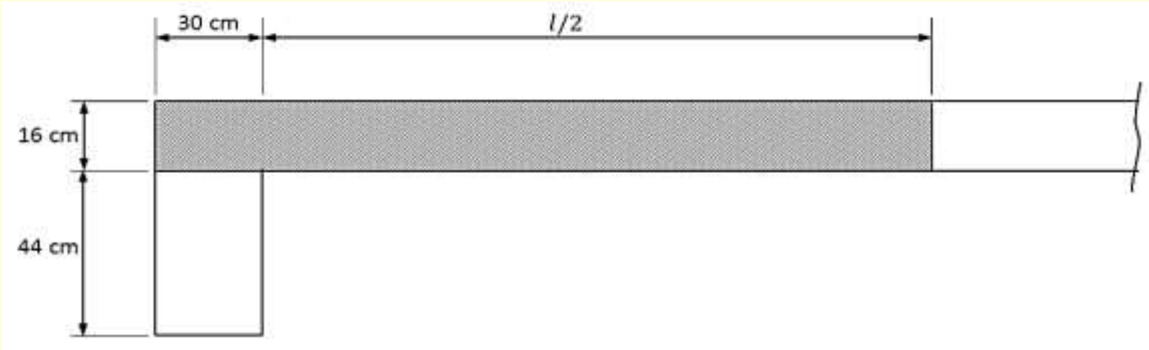
$b_w + 8h = 30 + 8(16) = 158\ cm$

$b_w + 2h_w = 118\ cm < \ b_w + 8h = 158\ cm\ OK$

$$y_c = \frac{16(30 + 44 \times 2)\left(44 + \frac{16}{2}\right) + 30 \times 44 \times \frac{44}{2}}{16(30 + 44 \times 2) + 30 \times 44} = 39.7\ cm$$

$$I_b = \frac{(30 + 44 \times 2)(16 + 4.3)^3}{3} - \frac{2 \times 44 \times 4.3^3}{3} + 30\frac{39.7^3}{3}$$
$$= 952416\ cm^4$$

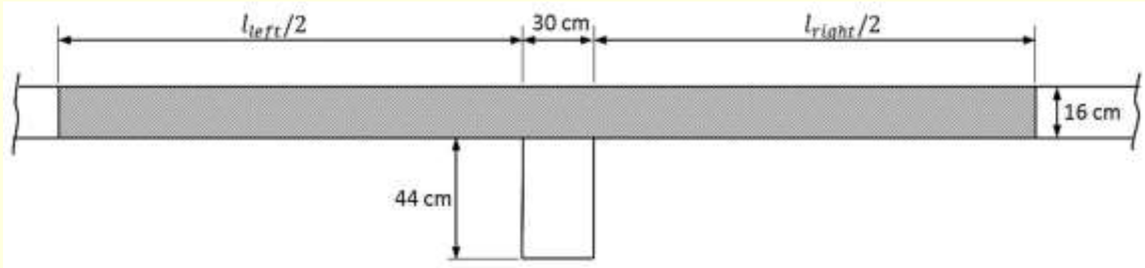
- Slab section for Exterior beam:



Short direction $l = 6.2\ m = 620\ cm, I_s = \frac{\left(\frac{620}{2}+30\right)16^3}{12} = 116053\ cm^4$

Long direction $l = 7.7\ m = 770\ cm, I_s = \frac{\left(\frac{770}{2}+30\right)16^3}{12} = 141653\ cm^4$

- Slab section for Interior beam:



Short direction

$$l_{right} = l_{left} = 6.2\ m = 620\ cm, I_s = \frac{(620+30)16^3}{12} = 221867\ cm^4$$

Long direction

$$l_{right} = l_{left} = 7.7\ m = 770\ cm, I_s = \frac{(770+30)16^3}{12} = 273067\ cm^4$$

$$\alpha_f = \frac{E_{cb}I_b}{E_{cs}I_s} = \frac{I_b}{I_s}\ \ where\ \ E_{cb} = E_{cs}$$

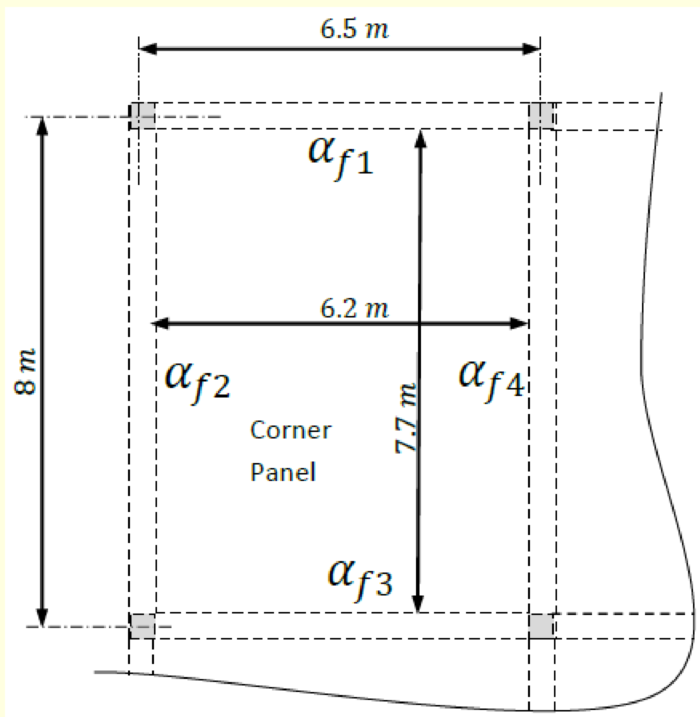
$$\alpha_{f1} = \frac{I_b}{I_s} = \frac{799957}{141653} = 5.6$$

$$\alpha_{f2} = \frac{I_b}{I_s} = \frac{799957}{116053} = 6.9$$

$$\alpha_{f3} = \frac{I_b}{I_s} = \frac{799957}{273067} = 3.5$$

$$\alpha_{f4} = \frac{I_b}{I_s} = \frac{799957}{221867} = 4.3$$

$$\alpha_{fm} = \frac{\sum \alpha_f}{4} = \frac{5.6+6.9+3.5+4.3}{4} = 5.1 > 2.0$$



$\alpha_{fm} = 5.1 > 2.0$ *the minimum slab thickness will be*:

$$\beta = \frac{l_{n,long}}{l_{n,short}} = \frac{7.7}{6.2} = 1.24$$

$$h = \frac{l_n\left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta} = \frac{7700\left(0.8 + \frac{400}{1400}\right)}{36 + 9 \times 1.24} = 177mm > 90mm\ OK$$

First trial thickness $h = 160mm < 177mm$.

Take slab thickness $h_{slab} = 200\ mm$.

2. Loads calculation:

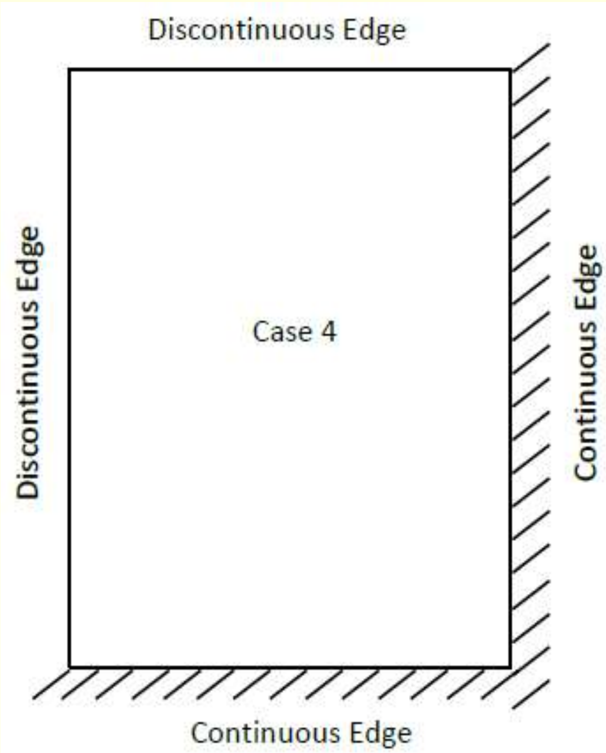
Material	Quality Density $KN/\,m^3$	$W = \gamma \cdot h$ $KN/\,m^2$
Tiles	22	$22 \times 0.03 = 0.66$
mortar	22	$22 \times 0.02 = 0.44$
Sand	16	$16 \times 0.07 = 1.12$
Reinforced Concrete solid slab	25	$25 \times 0.2 = 5$
Plaster	22	$22 \times 0.02 = 0.44$
Partitions $2\ KN/\,m^2$		2
Total Dead Load		9.66

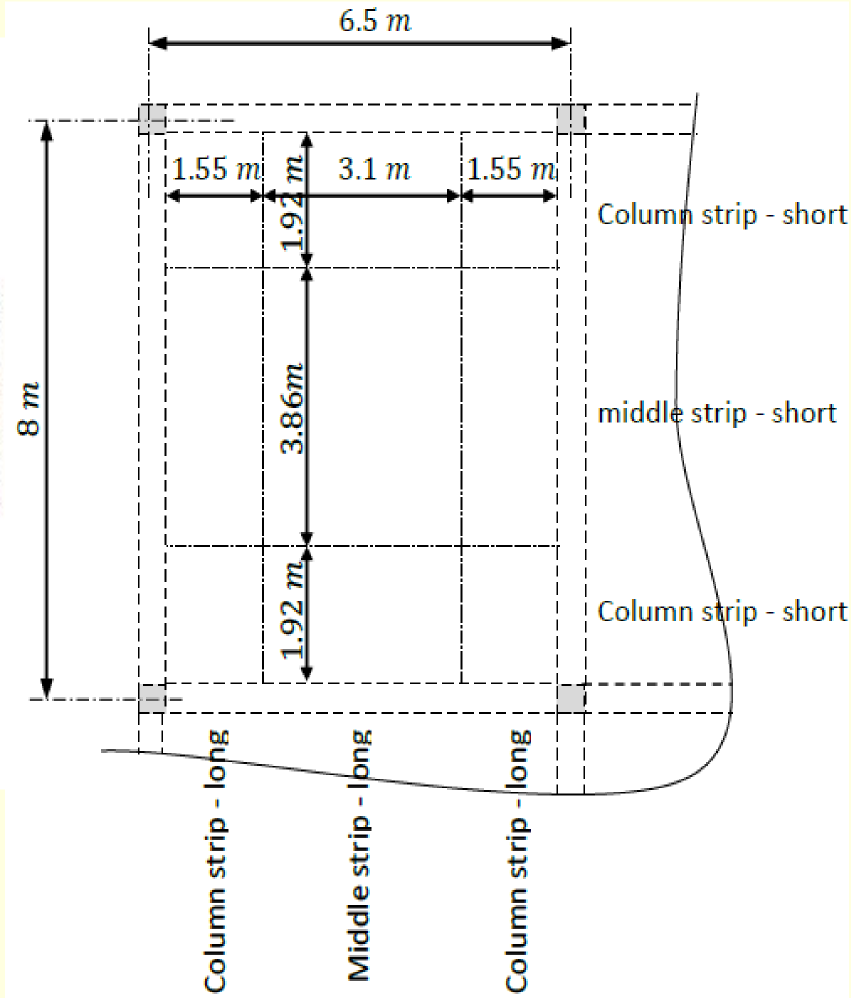
Dead Load of slab $DL = 9.66\ KN/m^2$, $w_D = 1.2 \cdot 9.66 = 11.6\ KN/m^2$

Live Load of slab $LL = 5\ KN/m^2$, $w_L = 1.6 \cdot 5 = 8\ KN/m^2$

$$w = 11.6 + 8 = 19.6\ KN/m^2$$

3. Moments calculations:





$$M_a = C_a w l_a^2 \qquad M_b = C_b w l_b^2$$

$$\frac{l_a}{l_b} = \frac{6.2}{7.7} = 0.81$$

The moment calculation will be done for the slab middle strip.

- Negative moments at continuous edge (table1):

$$C_{a,neg.}\left(\frac{l_a}{l_b} = 0.8\right) = 0.071 \; and \; C_{a,neg.}\left(\frac{l_a}{l_b} = 0.85\right) = 0.066$$

$$C_{a,neg.}\left(\frac{l_a}{l_b} = 0.81\right) = 0.071 - \left(\frac{0.071 - 0.066}{0.85 - 0.8}\right)(0.81 - 0.8) = 0.07$$

$$M_{a,neg.} = 0.07 \times 19.6 \times 6.2^2 = 52.7 kN.\frac{m}{m}$$

$$C_{b,neg.}\left(\frac{l_a}{l_b}=0.8\right)=0.029\ and\ C_{b,neg.}\left(\frac{l_a}{l_b}=0.85\right)=0.034$$

$$C_{b,neg.}\left(\frac{l_a}{l_b}=0.81\right)=0.029-\left(\frac{0.034-0.029}{0.85-0.8}\right)(0.81-0.8)=0.03$$

$$M_{b,neg.}=0.03\times 19.6\times 7.7^2=34.9kN.\frac{m}{m}$$

- Positive moments (Table 2 and Table 3):

$$C_{a,D}\left(\frac{l_a}{l_b}=0.8\right)=0.039\ and\ C_{a,D}\left(\frac{l_a}{l_b}=0.85\right)=0.036$$

$$C_{a,D}\left(\frac{l_a}{l_b}=0.81\right)=0.039-\left(\frac{0.039-0.036}{0.85-0.8}\right)(0.81-0.8)=0.0384$$

$$M_{a,pos,D}=0.0384\times 11.6\times 6.2^2=17.1kN.\frac{m}{m}$$

$$C_{a,L}\left(\frac{l_a}{l_b}=0.8\right)=0.048\ and\ C_{a,L}\left(\frac{l_a}{l_b}=0.85\right)=0.043$$

$$C_{a,L}\left(\frac{l_a}{l_b}=0.81\right)=0.048-\left(\frac{0.048-0.043}{0.85-0.8}\right)(0.81-0.8)=0.047$$

$$M_{a,pos,L}=0.047\times 8\times 6.2^2=14.5kN.\frac{m}{m}$$

$$M_{a,pos}=M_{a,pos,D}+M_{a,pos,L}=17.1+14.5=31.6kN.\frac{m}{m}$$

$$C_{b,D}\left(\frac{l_a}{l_b}=0.8\right)=0.016\ and\ C_{b,D}\left(\frac{l_a}{l_b}=0.85\right)=0.019$$

$$C_{b,D}\left(\frac{l_a}{l_b}=0.81\right)=0.016-\left(\frac{0.019-0.016}{0.85-0.8}\right)(0.81-0.8)=0.0166$$

$$M_{b,pos,D}=0.0166\times 11.6\times 7.7^2=11.4kN.\frac{m}{m}$$

$$C_{b,L}\left(\frac{l_a}{l_b}=0.8\right)=0.020\ and\ C_{b,L}\left(\frac{l_a}{l_b}=0.85\right)=0.023$$

$$C_{b,L}\left(\frac{l_a}{l_b}=0.81\right)=0.02-\left(\frac{0.023-0.020}{0.85-0.8}\right)(0.81-0.8)=0.023$$

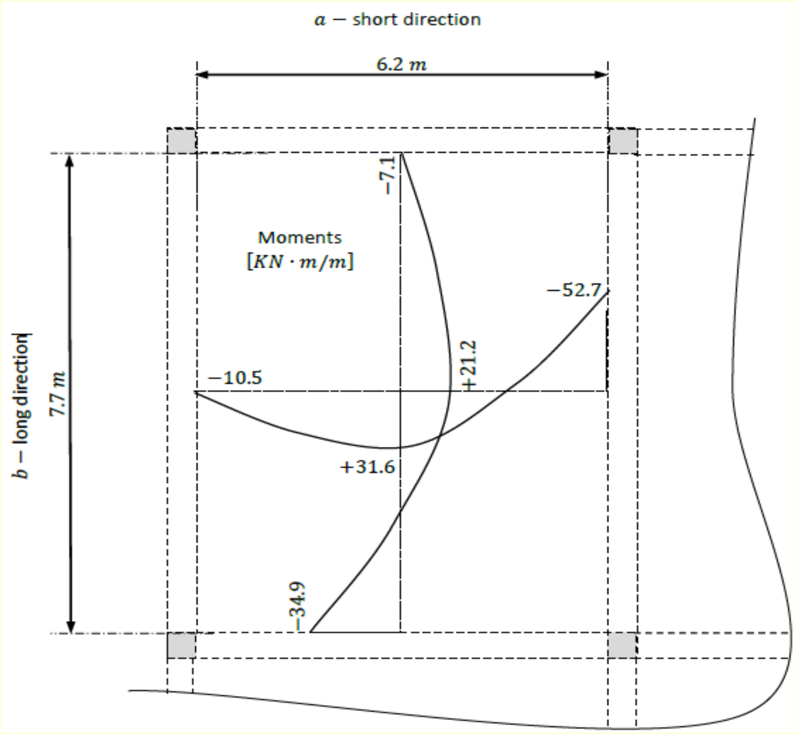
$$M_{b,pos,L}=0.023\times 8\times 7.7^2=9.8kN.\frac{m}{m}$$

$$M_{b,pos}=M_{b,pos,D}+M_{b,pos,L}=11.4+9.8=21.2\ kN.\frac{m}{m}$$

- Negative moments at Discontinuous edges ($\frac{1}{3}\times positive\ moments$):

$$M_{a,neg.}=\frac{1}{3}\times 31.6$$
$$=10.5\ kN.m/m$$

$$M_{b,neg.}=\frac{1}{3}\times 21.2$$
$$=7.1\ kN.m/m$$



4. Slab reinforcement:

- Short direction:

Assume bar diameter $\emptyset 10$ for main reinforcement.

$$d=h-20-\frac{d_b}{2}=200-20-\frac{10}{2}=175\ mm$$

- Midspan:

$$M_n = \frac{M_u}{\emptyset} = \frac{31.6}{0.9} = 35.1kN.m/m$$

$R_n = \frac{M_n}{bd^2} = \frac{35.1\times10^6}{1000\times175^2}$=1.15MPa

$$m = \frac{f_y}{085f'_c} = \frac{400}{0.85\times20} = 23.53$$

$$\rho = \frac{1}{m}\left(1-\sqrt{1-\frac{2R_nm}{f_y}}\right) = \frac{1}{23.53}\left(1-\sqrt{1-\frac{2\times1.15\times23.53}{400}}\right)$$
$$= 0.00298$$

$A_s = \rho bd = 0.00298\times1000\times175 = 521.5mm^2$

$A_{s,min} = \rho_{min}bd = 0.0018\times1000\times175 = 360\ mm^2$

$A_s = 521.5mm^2 > A_{s,min} = 360\ mm^2\ \ OK\ provide\ A_s = 521.5mm^2$

Use $\emptyset10\ then:$

$$NO.of\ bars = \frac{A_s}{A_{s\emptyset10}} = \frac{521.5}{79} = 6.6$$

$$the\ step\ of\ main\ reinforcement\ S = \frac{1}{n} = \frac{1}{6.6} = 0.152m$$

$$use\ \emptyset\ 10\ @\ 150\ mm, or\ use\ 7\emptyset\frac{10}{m}$$

$S = 150\ mm < 2h = 2\times200 = 400mm < 450mm\ \ OK$

Note that in the edge strips the positive moment, and the corresponding steel reinforcement area, is assumed to decrease from its full value at the edge of the middle strip to one-third of this value at the edge of the panel, which will not be provided.

- **Continuous edge:**

Assume bar diameter Ø 14 for main reinforcement.

$$d = h - 20 - \frac{d_b}{2} = 200 - 20 - \frac{14}{2} = 173\ mm$$

$$M_n = \frac{M_u}{\emptyset} = \frac{52.7}{0.9} = 58.6 kN.m/m$$

$$R_n = \frac{M_n}{bd^2} = \frac{58.6\times10^6}{1000\times173^2}$$=1.96MPa

$$m = \frac{f_y}{085 f'_c} = \frac{400}{0.85 \times 20} = 23.53$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2R_n m}{f_y}}\right) = \frac{1}{23.53}\left(1 - \sqrt{1 - \frac{2 \times 1.96 \times 23.53}{400}}\right)$$
$$= 0.00522$$

$$A_s = \rho bd = 0.00522 \times 1000 \times 173 = 903 mm^2$$

$$A_{s,min} = \rho_{min} bd = 0.0018 \times 1000 \times 175 = 360\ mm^2$$

$$A_s = 903mm^2 > A_{s,min} = 360\ mm^2\ \ OK\ provide\ A_s = 903\ mm^2$$

Use Ø14 $then$:

$$NO. of\ bars = \frac{A_s}{A_{s\emptyset 14}} = \frac{521.5}{153.9} = 5.9$$

$$the\ step\ of\ main\ reinforcement\ S = \frac{1}{n} = \frac{1}{5.9} = 0.170m$$

$$use\ \emptyset\ 14\ @\ 170\ mm, or\ use\ 6\emptyset\frac{14}{m}$$

$$S = 170\ mm < 2h = 2 \times 200 = 400mm < 450mm\ \ \ OK$$

- **Discontinuous edge**.

The negative moment at the discontinuous edge is one-third the positive moment in the span.

$$A_s \frac{1}{3} A_{s,pos} = \frac{1}{3} 521.5 = 173.8mm^2 < A_{s,min} = 360mm^2 - not\ ok$$

$$provide\ A_{s,min} = 360mm^2$$

Use $\emptyset 10 then$:

$$NO.of\ bars = \frac{A_s}{A_{s\emptyset 10}} = \frac{360}{78.54} = 4.9$$

$$the\ step\ of\ main\ reinforcement\ S = \frac{1}{n} = \frac{1}{4.9} = 0.205m$$

$$use\ \emptyset\ 10\ @\ 200\ mm, or\ use\ 5\emptyset \frac{10}{m}$$

$$S = 200\ mm < 2h = 2 \times 200 = 400mm < 450mm \quad OK$$

- **Long direction.**

H.W-Design for positive and negative moment as in the short direction. Note that the effective depth for the long direction will be:

$$d = h - 20 - d_{b,in\ the\ short\ direction} - \frac{d_b}{2}$$

5. Check for shear:

$$W_a\left(\frac{l_a}{l_b} = 0.8\right) = 0.71\ and\ W_a\left(\frac{l_a}{l_b} = 0.85\right) = 0.66$$

$$W_a\left(\frac{l_a}{l_b} = 0.81\right) = 0.71 - \left(\frac{0.71 - 0.66}{0.85 - 0.8}\right)(0.81 - 0.8) = 0.70$$

$$W_b\left(\frac{l_a}{l_b}=0.8\right)=0.29\ and\ W_b\left(\frac{l_a}{l_b}=0.85\right)=0.34$$
$$W_b\left(\frac{l_a}{l_b}=0.81\right)=0.29+\left(\frac{0.34-0.29}{0.85-0.8}\right)(0.81-0.8)=0.30$$

The reactions of the slab are calculated from Table 4, which indicates that 70% of the load is transmitted in the short direction and 30% in the long direction.

- The total load on the panel being $(6.2\times 7.7\times 19.6=935.7kN)$
- The load per meter on the long beam is $\left(0.7\times\frac{935.7}{2\times 7.7}\right)=42.5kN/m$
- The load per meter on the short beam is $\left(0.3\times\frac{935.7}{2\times 6.2}\right)=22.6kN/m$.

The shear to be transmitted by the slab to these beams is numerically equal to these beam loads, reduced to a critical section a distance d from the beam face. The shear strength of the slab is

$$V_c=\frac{1}{6}\lambda\sqrt{f'_c}b_wd=\frac{1}{6}\times 1\times\sqrt{20}\times 1000\times 173\times 10^{-3}=129kN$$

$$\emptyset V_c=0.75\times 129=96.75kN$$

$$V_{u,max}=42.5\times 1m=42.5\ kN<\frac{1}{2}\emptyset V_c=\frac{96.75}{2}=48.4\ kN$$

The thickness of the slab is adequate enough.
$V_{u,max}=42.5\ kN$ at the face of support, V_u at distance d from the face of support will be smaller.
Even, if $\frac{1}{2}\emptyset V_c<V_u\leq\emptyset V_c$ for solid slabs, the thickness of the slab will be enough.